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NONEXISTENCE OF CERTAIN Q-POLYNOMIAL DISTANCE-REGULAR GRAPHS

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I. N. Belousov, A. A. Makhnev, and M. S. Nirova described Q-polynomial distance-regular graphs Γ of diameter 3 for which the graphs Γ_2 and Γ_3 are strongly regular. Set $a = a_3$. A graph Γ has type (I) if c_2+1 divides a, type (II) if c_2+1 divides $a+1$, and type (III) if c_2+1 divides neither a nor $a+1$. If Γ is a graph of type (II), then $a+1 = w(c_2+1)$, $t^2 = w(w(c_2+1)+c_2)$, and either

(i) $w = s^2$, $t^2 = s^2(s^2(c_2+1)+c_2)$, $(s^2(c_2+1)+c_2$ is the square of an integer u, $c_2 = (u^2-s^2)/(s^2+1)$, $t = su$, and $a = (u^2s^2-1)/(s^2+1)$ or

(ii) $c_2 = sw$, $t^2 = w^2(sw+1+s)$, $sw+1+s$ is the square of an integer u, $c_2 = (u^2-1)w/(w+1)$, $t = uw$, $a = (u^2w^2-1)/(w+1)$, and Γ has intersection array

$$\left\{\frac{u^3w^2+u^2w^2+uw-1}{w+1}, \frac{(u^2-1)uw^2}{w+1}, \frac{(u^2w+1)w}{w+1}; 1, \frac{(u^2-1)w}{w+1}, \frac{(u^2w+1)uw}{w+1}\right\}.$$

If a graph of type (IIii) is such that $w = u$, then it has intersection array $\{w^4+w-1, w^4-w^3, (w^2-w+1)w; 1, w(w-1), (w^2-w+1)w^2\}$. We prove that graphs with such intersection arrays do not exist for even w.

Keywords: distance-regular graph, Q-polynomial graph.

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