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SEMIFIELD PLANES OF RANK 2 ADMITTING THE GROUP S_3

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One of the classical problems in projective geometry is to construct an object from known constraints on its automorphisms. We consider finite projective planes coordinatized by a semifield, i.e., by an algebraic system satisfying all axioms of a skew-field except for the associativity of multiplication. Such a plane is a translation plane admitting a transitive elation group with an affine axis. Let π be a semifield plane of order p^{2n} with a kernel containing $GF(p^n)$ for prime p, and let the linear autotopism group of π contain a subgroup H isomorphic to the symmetric group S_3. For the construction and analysis of such planes, we use a linear space and a spread set, which is a special family of linear mappings. We find a matrix representation for the subgroup H and for the spread set of a semifield plane if $p = 2$ and if $p > 2$. We also study the existence of central collineations in H. It is proved that a semifield plane of order 3^{2n} with kernel $GF(3^n)$ admits no subgroups isomorphic to S_3 in the linear autotopism group. Examples of semifield planes of order 16 and 625 admitting S_3 are found. The obtained results can be generalized for semifield planes of rank greater than 2 and can be applied, in particular, for studying the known hypothesis that the full collineation group of any finite non-Desarguesian semifield plane is solvable.

Keywords: semifield plane, autotopism group, symmetric group, Baer involution, homology, spread set.

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