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QUESTIONS OF THE STRUCTURE OF FINITE NEAR-FIELDS

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A semifield is a simple ring in which nonzero elements with respect to multiplication form a loop. Weakening distributivity from two-sided to one-sided yields the more general notion of quasifield (near-field under the condition of associativity). Problems of the structure of finite semifields and quasifields have been studied in various cases for a long time. In recent years, they have been mentioned in a number of papers. These problems were solved earlier for Knuth–Rúa and Hentzel–Rúa semifields, which are counterexamples of orders 32 and 64 to Wene's known hypothesis. The methods of computer algebra were used to describe some quasifields of small orders. It is known that the center of a finite semifield always contains the prime subfield. We show that the center of a finite near-field Q contains the prime subfield P except for Zassenhaus' four near-fields of orders 5^2, 7^2, 11^2, and 29^2. The kernel of a near-field Q always contains P. The maximal subfields of a finite near-field are enumerated under sufficiently general conditions. The automorphism groups of a near-field Q and of its multiplicative group Q^* were found earlier. The group Q^* is metacyclic, which makes it possible to explicitly find the spectrum of group orders of its elements.

Keywords: quasifield, semifield, near-field, maximal subfield, spectrum.

REFERENCES

1. Kurosh A.G. *Lectures on general algebra.* International Ser. Monographs on Pure and Applied Math., vol. 70, N Y Inc.: Elsevier Ltd., 1965, 374 p. doi: 10.1016/C2013-0-01775-6 . Original Russian text published in Kurosh A.G. *Lektsii po obshchei algebre.* Moscow: Fizmatgiz Publ., 1962, 396 p.
2. Dickson L.E. Linear algebras in which division is always uniquely possible. *Trans. Amer. Math. Soc.*, 1906, vol. 7, no. 3, pp. 370–390. doi: 10.2307/1986324 .
3. Veblen O., Maclagan-Wedderburn J.H. Non-desarguesian and Non-pascalian geometries. *Trans. Amer. Math. Soc.*, 1907, vol. 8, no. 3, pp. 379–388. doi: 10.2307/1988781 .
4. Hughes D.R., Piper F.C. *Projective planes.* N Y: Springer-Verlag, 1973, 292 p. ISBN: 0387900446 .
5. Johnson N.L., Jha V., Biliotti M. *Handbook of finite translation planes.* London; N Y: Chapman Hall/CRC, 2007, 888 p. ISBN: 1420011146 .
6. Levchuk V.M., Kravtsova O.V. Problems on structure of finite quasifields and projective translation planes. *Lobachevskii J. Math.*, 2017, vol. 38, no. 4, pp. 688–698. doi: 10.1134/S1995080217040138 .
7. Wene G.P. On the multiplicative structure of finite division rings. *Aeq. Math.*., vol. 41, no. 1, pp. 222–233. doi: 10.1007/BF02227457 .
8. Zassenhaus H. Über endliche Fastkörper. *Abh. Math. Semin. Univ. Hambg.*, vol. 11, no. 1, pp. 187–220. doi: 10.1007/BF02940723 .
9. Hall M. *The theory of groups.* N Y: Chelsea Pub. Co., 1976, 434 p. ISBN: 0828402884 . Translated to Russian under the title *Teoriya grupp.* Moscow: Izd. Inostr. Lit., 1962, 468 p.
10. Yakovleva T.N. Questions of construction of quasifields with associative powers. *Izv. Irkutsk. Gos. Univ., Ser. Mat.*, 2019, vol. 29, pp. 107–119 (in Russian). doi: 10.26516/1997-7670.2019.29.107 .
11. Felgner U. Pseudo-finite near-fields. In C. Betrh (ed.), *Near-rings and near-fields*, Ser. North-Holland Mathematics Studies, vol. 137, 1987, pp. 15–29. doi: 10.1016/S0304-0208(08)72282-5 .
12. Coxeter H.S.M., Moser W.O.J. *Generators and relations for discrete groups.* Berlin: Springer Verlag, 1972, 164 p. doi: 10.1007/978-3-662-21946-1 . Translated to Russian under the title *Porozhdayushchie elementy i opredelyayushchie sootnosheniya diskretnykh grupp.* Moscow: Nauka Publ., 1980, 240 p.
13. Dancs S. The sub-near-field structure of finite near-fields. *Bull. Austral. Math. Soc.*, 1971, vol. 5, pp. 275–280. doi: 10.1017/S000497270004716X .

14. Dancs Groves S. Locally finite near-fields. *Abh. Math. Sem. Univ. Hamburg*, 1979, vol. 48, pp. 89–107. doi: 10.1017/S0004972700043914 .
15. Dancs S. On finite Dickson near-fields. *Abh. Math. Sem. Univ. Hamburg*, 1972, vol. 37, no. 3-4, pp. 254–257. doi: 10.1007/BF02999702 .
16. Ellers E., Karzel H. Endliche Inzidenzgruppen. *Abh. Math. Semin. Univ. Hambg.*, vol. 27, no. 3-4, pp. 250–264. doi: 10.1007/BF02993220 .
17. Wähling H. *Theorie der Fastkörper.* Vol. 1 of Thales Monographs. Essen: Thales-Verlag, 1987. 393 p.
18. Boykett T., Howell K.T. The multiplicative automorphisms of a finite nearfield, with an application. *Commun. Algebra*, 2016, vol. 44, no. 6, pp. 2336–2350. doi: 10.1080/00927872.2015.1044105 .

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