TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 25 No. 3 **2019**

MSC: 65J20, 68U10

DOI: 10.21538/0134-4889-2019-25-3-9-23

ON THE LOCALIZATION OF NONSMOOTH DISCONTINUITY LINES OF A FUNCTION OF TWO VARIABLES

L. Ageev, T. V. Antonova

We consider ill-posed problems of localizing (finding the position of) the discontinuity lines of a perturbed function of two variables (an image). For each node of a uniform square grid with step τ, the average values of the function over a square $\tau \times \tau$ are assumed to be known. The perturbed function approximates an exact function in the space $L_2(\mathbb{R}^2)$, and the perturbation level δ is known. Earlier, the authors studied the case of piecewise smooth discontinuity lines, which, as a rule, correspond to the borders of artificial objects in the corresponding image. In the present paper, an approach to the study of localization algorithms is developed, which makes it possible to weaken the conditions on the smoothness of discontinuity lines and consider, in particular, nonsmooth discontinuity lines, which can describe the boundaries of natural objects. To solve the problem under consideration, we construct and analyze global discrete algorithms for the approximation of discontinuity lines by sets of points of a uniform grid on the basis of averaging procedures. Conditions on the exact function are formulated and a correctness class is constructed, which includes functions with nonsmooth discontinuity lines. A theoretical analysis of the constructed algorithms is carried out on this class. It is established that the proposed algorithms make it possible to obtain a localization error of order $O(\delta)$. We also estimate other important parameters, which characterize the operation of the localization algorithm.

Keywords: ill-posed problem, regularization method, discontinuity lines, global localization, discretization, separability threshold.

REFERENCES

1. Tikhonov A.N., Arsenin V.Ya. *Solutions of ill-posed problems*. New York etc.: John Wiley & Sons, 1977, 258 p. ISBN: 0-470-99124-0 . Original Russian text published in Tikhonov A.N., Arsenin V.Ya. *Metody resheniya nekorrektnykh zadach*. Moscow: Nauka Publ., 1974, 223 p.
2. Ivanov V.K., Vasin V.V., Tanana V.P. *Theory of linear ill-posed problems and its applications*. Inverse and Ill-Posed Problems Series. Utrecht: VSP, 2002, 281 p. ISBN: 90-6764-367-X/hbk . Original Russian text published in Ivanov V.K., Vasin V.V., Tanana V.P. *Teoriya lineinykh nekorrektnykh zadach i ee prilozheniya*. Moscow: Nauka Publ., 1978, 208 p.
3. Vasin V.V., Ageev A.L. Ill-posed problems with a priori information. Utrecht: VSP, 1995, 255 p.
4. Mallat S. *A wavelet tour of signal processing: the sparse way*. N Y: Acad. Press, 1999, 620 p. ISBN: 0-12-466606-X . Translated to Russian inder the title Malla S. *Veivlety v obrabotke signalov*. Moscow: Mir Publ., 2005, 671 p.
5. Furman Ya.A. (ed.). *Vvedenie v konturnyi analiz i ego prilozheniya k obrabotke izobrazhenii i signalov* [Introduction to Contour Analysis and its Application to Image and Signal Processing]. Moscow: Fizmatlit Publ., 2002, 596 p. ISBN: 5-9221-0255-9 .
6. Gonzalez R.C., Woods R.E. *Digital image processing (3rd Edition)*. N J: Pearson Prentice Hall, 2006, 976 p. ISBN: 978-0131687288 . Translated to Russian inder the title *Tsifrovaya obrabotka izobrazhenii. (Izdanie 3-e ispravlennoe i dopolnennoe)*. Moscow: Tekhnosfera, 2012, 1104 p.
7. Antonova T.V. A method for localization of discontinuity lines of an approximately defined function of two variables. *Numerical Anal. Appl.*, 2012, vol. 5, no. 4, pp. 285–296. doi: 10.1134/S1995423912040015 .
8. Ageev A.L., Antonova T.V. Approximation of discontinuity lines of a noisy function of two variables. *J. Appl. Industrial Math.*, 2012, vol. 6, no. 3, pp. 269–279. doi: 10.1134/S1990478912030015 .
9. Ageev A.L., Antonova T.V. A Discrete Algorithm for Localizing the Discontinuity Lines of a Function of Two Variables. *J. Appl. Industrial Math.*, 2017, vol. 11, no. 4, pp. 463–471. doi: 10.1134/S1990478917040019 .

10. Ageev A.L., Antonova T.V. On the problem of global localization of discontinuity lines for a function of two variables. *Trudy Inst. Mat. i Mekh. UrO RAN*, 2018, vol. 24, no. 2, pp. 12–23 (in Russian). doi: 10.21538/0134-4889-2018-24-2-12-23 .

Received June 11, 2019
Revised July 22, 2019
Accepted July 29, 2019

Aleksandr Leonidovich Ageev, Dr. Phys.-Math. Sci., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia; Ural Federal University, Yekaterinburg, 620083 Russia, e-mail: ageev@imm.uran.ru .

Tat'yana Vladimirovna Antonova, Dr. Phys.-Math. Sci., Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia, e-mail: tvantonova@imm.uran.ru .

Cite this article as: A. L. Ageev, T. V. Antonova. On the localization of nonsmooth discontinuity lines of a function of two variables, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2019, vol. 25, no. 3, pp. 9–23 .