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ASYMPTOTIC BEHAVIOR OF REACHABLE SETS ON SMALL TIME INTERVALS

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The geometric structure of small-time reachable sets plays an important role in control theory, in particular, in solving problems of local synthesis. In this paper, we consider the problem of approximate description of reachable sets on small time intervals for control-affine systems with integral quadratic constraints on the control. Using a time substitution, we replace such a set by the reachable set on a unit interval of a control system with a small parameter, which is the length of the time interval for the original system. The constraints on the control are given by a ball of small radius in the Hilbert space $\mathbb{L}_2$. Under certain conditions imposed on the controllability Gramian of the linearized system, this reachable set turns out to be convex for sufficiently small values of the parameter. We show that in this case the shape of the reachable set in the state space is asymptotically close to an ellipsoid. The proof of this fact is based on the representation of the reachable set as the image of a Hilbert ball of small radius in $\mathbb{L}_2$ under a nonlinear mapping to $\mathbb{R}^n$. In particular, this asymptotic representation holds for a fairly wide class of second-order nonlinear control systems with integral constraints. We give three examples of systems whose reachable sets demonstrate both the presence of the indicated asymptotic behavior and the absence of the latter if the necessary conditions are not satisfied.

Keywords: control system, integral constraints, reachable set, convexity, asymptotics.

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