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ON A DIFFERENTIAL GAME IN A STOCHASTIC SYSTEM

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We study the game problem of approach for a system whose dynamics is described by a stochastic differential equation in a Hilbert space. The main assumption on the equation is that the operator multiplying the system state is the generator of a strongly continuous semigroup (a semigroup of class C_0). Solutions of the equation are represented by a stochastic formula of variation of constants. Using constraints on the support functionals of sets defined by the behavior of the pursuer and the evader, we obtain conditions for the approach of the system state to a cylindrical terminal set. The results are illustrated with a model example of a simple motion in a Hilbert space with random perturbations. Applications to distributed systems described by stochastic partial differential equations are considered. By taking into account a random external influence, we consider the heat propagation process with controlled distributed heat sources and leaks.

Keywords: differential game, stochastic differential equation, Wiener process, generator of a strongly continuous semigroup, set-valued mapping, support functional, resolving functional, stochastic partial differential equation.

REFERENCES

1. Krasovskii N.N. *Teoriya upravleniya dvizheniem. Lineinye sistemy* [Theory of motion control. Linear systems]. Moscow: Nauka Publ, 1968, 476 p.
2. Krasovskii N.N. *Igrovye zadachi o vstreche dvizhenii* [Game problems on the encounter of motions]. Moscow, Nauka Publ., 1970, 420 p.
3. Krasovskii N.N., Subbotin A.I. *Game-theoretical control problems*. N Y: Springer, 1988, 517 p. ISBN: 978-1-4612-8318-8 . Original Russian text published in Krasovskii N.N., Subbotin A.I. *Pozitsionnye differentsial'nye igry*. Moscow: Nauka Publ., 1974, 456 p.
4. Krasovskii N.N. *Upravlenie dinamicheskoi sistemoi* [Control of a dynamical system]. Moscow, Nauka Publ., 1985, 520 p.
5. Krasovskii N.N., Krasovskii A.N. *Control under lack of information*. Berlin etc.: Birkhäuser, 1995, 322 p. ISBN: 0-8176-3698-6 .
6. Osipov Yu.S., Kryazhimskii A.V. *Inverse problems for ordinary differential equations: dynamical solutions*. Basel: Gordon and Breach, 1995, 625 p.
7. Kurzhanskii A.B. *Upravlenie i nablyudenie v usloviyakh neopredelennosti* [Control and observation under the conditions of uncertainty]. Moscow: Nauka Publ., 1977, 392 p.
8. Subbotin A.I., Chentsov A.G. *Optimizatsiya garantii v zadachakh upravleniya* [Guarantee optimization in control problems]. Moscow: Nauka Publ., 1981, 288 p.
9. Krasovskii N.N. A convergence-evasion game with stochastic guide. *Dokl. Akad. Nauk SSSR*, 1977, vol. 237, no. 5, pp. 1020–1023. (in Russian)
10. Krasovskii N.N., Tret'yakov V.E. Saddle point of a stochastic differential game. *Soviet Math. Dokl.*, 1981, vol. 22, no. 2, pp. 393–398.
11. Krasovskii N.N., Tret'yakov V.E. Stochastic program synthesis for a positional differential game. *Soviet Math. Dokl.*, 1981, vol. 24, no. 1, pp. 17–20.
12. Krasovskii N.N. Deterministic strategy and stochastic programs. *J. Appl. Math. Mech.*, 1985, vol. 49, no. 2, pp. 135–143. doi: 10.1016/0021-8928(85)90092-9 .
13. Krasovskii N.N., Kotel'nikova A.N. Stochastic guide for a time-delay object in a positional differential game. *Proc. Steklov Inst. Math.*, 2012, vol. 277, suppl. 1, pp. 145–151. doi: 10.1134/S0081543812050148 .
14. Ramachandran K.M., Tsokos C.P. *Stochastic differential games*. Paris; Amsterdam; Beijing: Atlantis Press, 2012, 248 p. ISBN: 978-94-91216-47-3 .

15. Fleming W.H., Nisio M. Differential games for stochastic partial differential equations. *Nagoya Math. J.*, 1993, vol. 131, pp. 75–107. doi: 10.1017/S0027763000004554 .
16. Vlasenko L.A., Chikrii A.A. The method of resolving functionals for a dynamic game in a Sobolev system. *J. Automat. Inform. Sci.*, 2014, vol. 46, no. 7, pp. 1–11. doi: 10.1615/JAutomatInfScien.v46.i7.10 .
17. Vlasenko L.A., Rutkas A.G., Chikrii A.A. On a differential game in an abstract parabolic system. *Proc. Steklov Inst. Math.*, 2016, vol. 293, suppl. 1, pp. 254–269. doi: 10.1134/S0081543816050229 .
18. Chikrii A.A. *Conflict-controlled processes*. Boston; London; Dordrecht: Kluwer Acad. Publ., 1997, 424 p. ISBN: 0-79234522-3 .
19. Chikrii A.A. An analytical method in dynamic pursuit games. *Proc. Steklov Inst. Math.*, 2010, vol. 271, pp. 69–85. doi: 10.1134/S0081543810040073 .
20. Hille E., Phillips R.S. *Functional analysis and semigroups*. Amer. Math. Soc. Colloq. Publ., vol. 31, Providence, RI: AMS, 1957, 808 p. ISBN: 978-0-8218-3395-7 . Translated to Russian under the title *Funktsional'nyi analiz i polugruppy*, Moscow: Inostrannaya Literatura Publ., 1962, 830 p.
21. Curtain R.F., Falb P.L. Stochastic differential equations in Hilbert space. *J. Diff. Eq.*, 1971, vol. 10, no. 3, pp. 412–430. doi: 10.1016/0022-0396(71)90004-0 .
22. Dalecky Yu.L., Fomin S.V. *Measures and differential equations in infinite-dimensional space*. Dordrecht: Kluwer, 1991, 337 p. ISBN: 978-94-010-5148-4 . Original Russian text published in Daletskii Yu.L., Fomin S.V. *Mery i differentsial'nye uravneniya v beskonechnomernykh prostranstvakh*. Moscow: Nauka Publ., 1983, 383 p.
23. Da Prato G., Zabchyk J. *Stochastic equations in infinite dimensions*. Cambridge: Cambridge University Press, 1992, 454 p. doi: 10.1017/CBO9780511666223 .
24. Vlasenko L.A., Rutkas A.G. On a differential game in a system described by an implicit differential-operator equation. *Diff. Eq.*, 2015, vol. 51, no. 6, pp. 798–807. doi: 10.1134/S0012266115060117 .
25. Pazy A. *Semigroups of linear operators and applications to partial differential equations*. N Y; Berlin; Heidelberg; Tokyo: Springer-Verlag, 1983, 279 p. doi: 10.1007/978-1-4612-5561-1 .
26. Vlasenko L.A., Rutkas A.G. Stochastic impulse control of parabolic systems of Sobolev type. *Diff. Eq.*, 2011, vol. 47, no. 10, pp. 1498–1507. doi: 10.1134/S0012266111100132 .
27. Vlasenko L.A., Rutkas A.G. Optimal control of a class of random distributed Sobolev type systems with aftereffect. *J. Automat. Inform. Sci.*, 2013, vol. 45, no. 9, pp. 66–76. doi: 10.1615/JAutomatInfScien.v45.i9.60 .
28. Aubin J.P., Frankowska H. *Set-valued analysis*. Boston: Birkhäuser, 1990, 461 p. ISBN: 0-8176-3478-9 .
29. Dorogovtsev A.Ya., IvasishenS.D., Kukush A.G. Asymptotic behavior of solutions of the heat-conduction equation with white noise in the right side. *Ukrain. Math. J.*, 1985, vol. 37, no. 1, pp. 10–15. doi: 10.1007/BF01056844 .
30. Weits E. A Stochastic heat equation for stationary freeway traffic flow. *Theory Probab. Appl.*, 1993, vol. 37, no. 1, pp. 185–188. doi: 10.1137/1137049 .

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