ANALYSIS OF A REGULARIZATION ALGORITHM
FOR A LINEAR OPERATOR EQUATION
CONTAINING A DISCONTINUOUS COMPONENT OF THE SOLUTION

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We study a linear operator equation that does not satisfy the Hadamard well-posedness conditions. It is assumed that the solution of the equation has different smoothness properties on different segments of its domain. More exactly, the solution is representable as the sum of a smooth and discontinuous components. The Tikhonov regularization method is applied for the construction of a stable approximate solution. In this method, the stabilizer is the sum of the Lebesgue norm and the smoothed BV-norm. Each of the functionals in the stabilizer depends only on one component and takes into account its properties. Convergence theorems are proved for the regularized solutions and their discrete approximations. It is shown that discrete regularized solutions can be found with the use of the Newton method and nonlinear analogs of α-processes.

Keywords: ill-posed problem, regularization method, discontinuous solution, total variation, discrete approximation.

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