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ALGORITHMS FOR THE CONSTRUCTION OF THIRD-ORDER LOCAL EXPONENTIAL SPLINES WITH EQUIDISTANT KNOTS

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We construct new local exponential splines with equidistant knots corresponding to a third-order linear differential operator $\mathcal{L}_3(D)$ of the form

$$\mathcal{L}_3(D) = (D-\beta)(D-\gamma)(D-\delta) \quad (\beta,\gamma,\delta \in \mathbb{R}).$$

We also establish upper order estimates for the error of approximation by these splines in the uniform metric on the Sobolev class of three times differentiable functions $W_\infty^{\mathcal{L}_3}$. In particular, for the differential operator $\mathcal{L}_3(D) = D(D^2-\beta^2)$, we give a general scheme for the construction of local splines with additional knots, which leads in one case to known shape-preserving splines and in another case to new local interpolation splines exact on the kernel of $\mathcal{L}_3(D)$.

Keywords: local exponential splines, linear differential operator, approximation, interpolation.

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