# APPROXIMATE SOLUTION OF AN INVERSE BOUNDARY VALUE PROBLEM FOR A SYSTEM OF DIFFERENTIAL EQUATIONS OF PARABOLIC TYPE AND ESTIMATION OF THE ERROR OF THIS SOLUTION

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We study the problem of finding a boundary condition in the heat equation for a hollow ball made of a composite material consisting of two homogeneous components. The Dirichlet condition is considered as boundary conditions inside the ball at $r = r_0$. In the inverse problem, the temperature inside the ball is assumed to be unknown on an infinite time interval. For finding it, the temperature of the heat flux at the media interface for $r = r_1$ is measured. We analyze the direct problem, which allows us to give a strict formulation of the inverse problem and determine the functional spaces in which it is natural to solve the inverse problem. Estimating the error of the approximate solution presents a major difficulty, which is dealt with in this paper by the method of projection regularization. Using this method, we find order-exact estimates.

Keywords: error estimation, modulus of continuity, Fourier transform, ill-posed problem.

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