ON THE CONSTRUCTION OF A NONANTICIPATING SELECTION OF A MULTIVALUED MAPPING

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We study the properties of multivalued mappings of general form with respect to the possibility of finding their nonanticipating selections. The property of nonanticipation is formulated for an arbitrary domain by specifying some family of "test" subsets. Sufficient conditions for the existence of a nonanticipating selection of a nonanticipating multivalued mapping are proposed: the values of the mapping must be nonempty compact sets, and the family of "test" subsets must be totally ordered by inclusion. We illustrate the results by showing their applicability to the pursuit–evasion differential game in the form of P. Varaya and J. Lin.

Keywords: quasistrategy, nonanticipation, selection, topology.

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