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MULTIPLE CAPTURE OF A GIVEN NUMBER OF EVADERS IN A PROBLEM WITH FRACTIONAL DERIVATIVES AND A SIMPLE MATRIX

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A problem of pursuing a group of evaders by a group of pursuers with equal capabilities of all the participants is considered in a finite-dimensional Euclidean space. The system is described by the equation

$$D^{(\alpha)} z_{ij} = a z_{ij} + u_i - v_j, \quad u_i, v_j \in V,$$

where $D^{(\alpha)} f$ is the Caputo fractional derivative of order α of the function f, the set of admissible controls V is strictly convex and compact, and a is a real number. The aim of the group of pursuers is to capture at least q evaders; each evader must be captured by at least r different pursuers, and the capture moments may be different. The terminal sets are the origin. Assuming that the evaders use program strategies and each pursuer captures at most one evader, we obtain sufficient conditions for the solvability of the pursuit problem in terms of the initial positions. Using the method of resolving functions as a basic research tool, we derive sufficient conditions for the solvability of the approach problem with one evader in some guaranteed time. Hall's theorem on a system of distinct representatives is used in the proof of the main theorem.

Keywords: differential game, group pursuit, multiple capture, pursuer, evader, fractional derivative.

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