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THE STRUCTURE OF THE REACHABLE SET FOR THE DUBINS CAR WITH A STRICTLY ONE-SIDED TURN

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We study the structure of a three-dimensional reachable set “at instant” of the nonlinear control system often called the “Dubins car.” A controlled vehicle moves in the plane with constant speed and bounded turning radius. We consider the case where the object can turn to one side only and the rectilinear motion is forbidden by given control constraints. Based on the Pontryagin maximum principle, we obtain variants of controls leading to the boundary of the reachable set. Sections of the three-dimensional reachable set along the angular coordinate are considered. The boundaries of such sections are described analytically in the form of sets of smooth arcs. The paper lists all possible options for the structure of the sections. Each arc is defined by a certain type of piecewise constant control satisfying the maximum principle. The strict convexity of the sections along the angular coordinate is proved, and the smoothness of the boundary of the sections is analyzed.

Keywords: Dubins car, strictly one-sided turn, structure of a three-dimensional reachable set, Pontryagin maximum principle, piecewise constant control, strict convexity of sections of a reachable set along the angular coordinate.

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