ESTIMATION OF REACHABLE SETS FROM ABOVE WITH RESPECT TO INCLUSION FOR SOME NONLINEAR CONTROL SYSTEM

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The study of reachable sets of controlled objects is an important research area in optimal control theory. Such sets describe in a rough form the dynamical possibilities of the objects, which is important for theory and applications. Many optimization problems for controlled objects use the reachable set $D(T)$ in their statements. In the study of properties of controlled objects, it is useful to have some constructive estimates of $D(T)$ from above with respect to inclusion. In particular, such estimates are helpful for the approximate calculation of $D(T)$ by the pixel method. In this paper we consider two nonlinear models of direct regulation known in the theory of absolute stability with a control term added to the right-hand side of the corresponding system of differential equations. To obtain the required upper estimates with respect to inclusion, we use Lyapunov functions from the theory of absolute stability. Note that the upper estimates for $D(T)$ are obtained in the form of balls in the phase space centered at the origin.

Keywords: reachable set, Lyapunov function, absolute stability, direct regulation.

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