ON THE CONSTRUCTION AND ESTIMATES OF THE CAUCHY MATRIX FOR SYSTEMS WITH AFTEREFFECT

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A linear functional differential system with aftereffect of general form is considered. Basic relations that define the Cauchy matrix — the kernel of integral representation to solutions of the Cauchy problem — are presented. The role of the Cauchy matrix in the study of a wide range of problems in the theory of functional differential systems, including control problems with respect to a given system of objective functionals and boundary value problems with general boundary conditions, is indicated. The efficiency of solving these problems depends essentially on the possibility of constructing a sufficiently exact approximation to the Cauchy matrix of the system. We propose an approach to the approximate construction of the Cauchy matrix that combines iterative procedures and algorithms for the construction of a rather accurate initial approximation based on a special approximation of parameters of the system. Error estimates are established for the resulting approximations.

Keywords: linear systems with aftereffect, representation of solutions, Cauchy matrix.

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