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ON THE DEFINITION OF UNIFORM COMPLETE OBSERVABILITY

E. K. Makarov, S. N. Popova

The classical definitions of uniform complete controllability and uniform complete observability were formulated by R. Kalman for systems with coefficients from the class $L_2^{\mathrm{loc}}(\mathbb{R})$. E. L. Tonkov proposed alternative dual definitions for systems with bounded measurable coefficients. For the theory of control of asymptotic invariants of differential systems, it is useful to study the properties of uniform complete controllability and observability for systems with arbitrary coefficients. We propose a definition of uniform complete observability on an arbitrarily given family of closed intervals of the real axis under the assumption that some spaces of controls and measured outputs of the system are defined on each of the intervals. Here we do not impose any constraints on the system apart from the requirement of the existence of solutions, their uniqueness, and extendability to the whole real axis. Some basic properties of the introduced notions are given. It is established that, in the general case, uniform complete controllability and uniform complete observability are not dual properties for linear systems. Sufficient conditions for the presence of such a duality are obtained. Similar results are formulated for the pair "identifiability — reachability."

Keywords: linear systems, uniform complete observability, uniform complete controllability.

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Evgenii Konstantinovich Makarov, Dr. Phys.-Math. Sci., Prof., Institute of Mathematics, National Academy of Sciences of Belarus, Minsk, 220072 Belarus, e-mail: jcm@im.bas-net.by .

Svetlana Nikolaevna Popova, Dr. Phys.-Math. Sci., Udmurt State University, Izhevsk, 426034 Russia, e-mail: udsu.popova.sn@gmail.com .

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