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# MINIMAL SUBMANIFOLDS OF SPHERES AND CONES

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Intersections of cones of index zero with spheres are investigated. Fields of the corresponding minimal manifolds are found. In particular, we consider the cone $\mathbb{K} = \{x_0^2 + x_1^2 = x_2^2 + x_3^2\}$. Its intersection with the sphere $\mathbb{S}^3 = \sum_{i=0}^{3} x_i^2$ is often called the Clifford torus $\mathbb{T}$, because Clifford was the first to notice that the metric of this torus as a submanifold of $\mathbb{S}^3$ with the metric induced from $\mathbb{S}^3$ is Euclidian. In addition, the torus $\mathbb{T}$ considered as a submanifold of $\mathbb{S}^3$ is a minimal surface. Similarly, it is possible to consider the cone $\mathcal{K} = \{\sum_{i=0}^{3} x_i^2 = \sum_{i=4}^{7} x_i^2\}$, often called the Simons cone because he proved that $\mathcal{K}$ specifies a single-valued nonsmooth globally defined minimal surface in $\mathbb{R}^8$ which is not a plane. It appears that the intersection of $\mathcal{K}$ with the sphere $\mathbb{S}^7$, like the Clifford torus, is a minimal submanifold of $\mathbb{S}^7$. These facts are proved by using the technique of quaternions and the Cayley algebra.

Keywords: minimal surface, gaussian curvature, quaternions, octonions (Cayley numbers), field of extremals, Weierstrass function.

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