# ON THE EXACTNESS OF THE INEQUALITY OF DIFFERENT METRICS FOR TRIGONOMETRIC POLYNOMIALS IN THE GENERALIZED LORENTZ SPACE

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We consider the generalized Lorentz space $L_{\psi,\tau}(\mathbb{T}^m)$ defined by some continuous concave function $\psi$ such that $\psi(0) = 0$. For two spaces $L_{\psi_1,\tau_1}(\mathbb{T}^m)$ and $L_{\psi_2,\tau_2}(\mathbb{T}^m)$ such that $\alpha_{\psi_1} = \underline{\lim}_{t\to 0}\psi_1(2t)/\psi_1(t) = \beta_{\psi_2} = \overline{\lim}_{t\to 0}\psi_2(2t)/\psi_2(t)$, we prove an order-exact inequality of different metrics for multiple trigonometric polynomials. We also prove an auxiliary statement for functions of one variable with monotonically decreasing Fourier coefficients in a trigonometric system. In this statement we establish a two-sided estimate for the norm of the function $f \in L_{\psi,\tau}(\mathbb{T})$ in terms of the series composed of the Fourier coefficients of this function.

Keywords: generalized Lorentz space, Jackson–Nikol'skii inequality, trigonometric polynomial.

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