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# ASYMPTOTICS OF THE SOLUTION TO A SINGULARLY PERTURBED TIME-OPTIMAL CONTROL PROBLEM WITH TWO SMALL PARAMETERS

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The paper continues the author's previous studies. We consider a time-optimal control problem for a singularly perturbed linear autonomous system with two independent small parameters and smooth geometric constraints on the control in the form of a ball

$$\begin{cases} \dot{x} = y, & x,\, y \in \mathbb{R}^2, \quad u \in \mathbb{R}^2, \\ \varepsilon^3 \dot{y} = Jy + u, & \|u\| \leqslant 1, \quad 0 < \varepsilon, \mu \ll 1, \\ x(0) = x_0(\varepsilon, \mu) = (x_{0,1}, \varepsilon^3 \mu \xi)^*, \quad y(0) = y_0, \\ x(T_{\varepsilon,\mu}) = 0, \quad y(T_{\varepsilon,\mu}) = 0, \quad T_{\varepsilon,\mu} \longrightarrow \min, \end{cases}$$

where

$$J = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

The main difference of this case from the systems with fast and slow variables studied earlier is that here the matrix $J$ at the fast variables is the second-order Jordan block with zero eigenvalue and, thus, does not satisfy the standard asymptotic stability condition. Continuing the research, we consider initial conditions depending on the second small parameter $\mu$. We derive and justify a complete asymptotic expansion in the sense of Erdelyi of the optimal time and optimal control with respect to the asymptotic sequence $\varepsilon^\gamma(\varepsilon^k + \mu^k)$, $0 < \gamma < 1$.

Keywords: optimal control, time-optimal control problem, asymptotic expansion, singularly perturbed problem, small parameter.

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