

MSC: 41A05, 41A15, 41A25

DOI: 10.21538/0134-4889-2019-25-2-67-74

## CONVERGENCE OF QUARTIC INTERPOLATION SPLINES

Yu. S. Volkov

The problem of interpolation by quartic splines according to Marsden's scheme is considered. It is shown that the calculation of an interpolating spline in terms of the coefficients of expansion of its second derivative in  $L_1$ -normalized quadratic B-splines yields a system of linear equations for the chosen parameters. The matrix of the system is pentadiagonal and has a column diagonal dominance, which makes it possible to efficiently calculate the required parameters and establish the convergence of the spline interpolation process according to Marsden's scheme for any function from the class  $C^1$  on an arbitrary sequence of grids without any constraints. In Marsden's scheme, it is assumed that a knot grid is given and the interpolation nodes are chosen strictly in the middle. The established results are transferred to the case of interpolation by quartic splines according to Subbotin's scheme (the node grid and knot grid are swapped). Here the system of equations for the coefficients of expansion of the third derivative in  $L_\infty$ -normalized B-splines has a diagonal dominance, and the interpolation process converges for any interpolated function from the class  $C^3$ .

Keywords: quartic splines, interpolation, convergence, diagonally dominant matrices.

### REFERENCES

1. Ahlberg J.H., Nilson E.N., Walsh J.L. Best approximation and convergence properties of higher-order spline approximations. *J. Math. Mech.*, 1965, vol. 14, no. 2, pp. 231–243.
2. Ahlberg J., Nilson E., Walsh J. *The theory of splines and their applications*. N Y: Acad. Press, 1967, 284 p. ISBN: 9781483222950. Translated to Russian under the title *Teoriya splainov i ee prilozheniya*. Moscow: Mir Publ., 1972, 316 p.
3. Stechkin S.B., Subbotin Yu.N. *Splainy v vychislitel'noi matematike* [Splines in computational mathematics]. Moscow: Nauka Publ., 1976, 248 p.
4. de Boor C. *A practical guide to splines*. N Y: Springer, 1978, 392 p. ISBN: 978-3540903567. Translated to Russian under the title *Prakticheskoe rukovodstvo po splainam*. Moscow: Radio i Svyaz' Publ., 1985, 304 p.
5. Zavalov Yu.S., Kvasov B.I., Miroshnichenko V.L. *Metody splajn funktsij* [Methods of spline-functions]. Moscow: Nauka Publ., 1980, 352 p.
6. de Boor C. On the convergence of odd-degree spline interpolation. *J. Approxim. Theory*, 1968, vol. 1, no. 4, pp. 452–463. doi: 10.1016/0021-9045(68)90033-6.
7. Volkov Yu.S. Totally positive matrices in the methods of constructing interpolation splines of odd degree. *Siberian Adv. Math.*, 2005, vol. 15, no. 4, pp. 96–125.
8. Volkov Yu.S. Interpolation by splines of even degree according to Subbotin and Marsden. *Ukrainian Math. J.*, 2014, vol. 66, no. 7, pp. 994–1012. doi: 10.1007/s11253-014-0990-z.
9. Ahlberg J.H., Nilson E.N. Convergence properties of the spline fit. *J. Soc. Indust. Appl. Math.*, 1963, vol. 11, no. 1, pp. 95–104. doi: 10.1137/0111007.
10. Subbotin Yu.N. Piecewise-polynomial (spline) interpolation. *Math. Notes*, 1967, vol. 1, no. 1, pp. 41–45. doi: 10.1007/BF01221723.
11. Marsden M.J. Quadratic spline interpolation. *Bull. Amer. Math. Soc.*, 1974, vol. 80, no. 5, pp. 903–906. ISBN: 10.1090/S0002-9904-1974-13566-4.
12. Volkov Yu.S. Necessary conditions for uniform convergence of interpolation splines of fourth and fifth degrees. *Vychisl. Sist.*, Novosibirsk, 1982, vol. 93, pp. 30–38 (in Russian).

Received March 1, 2019

Revised March 25, 2019

Accepted April 1, 2019

**Funding Agency:** This work was supported by the Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences (state contract no. 0314-2016-0013), and partially by the Russian Foundation for Basic Research and the German Research Foundation (project no. 19-51-12008).

*Yuriy Stepanovich Volkov*, Dr. Phys.-Math. Sci., Prof., Sobolev Institute of Mathematics of the Siberian Branch of the Russian Academy of Sciences, Novosibirsk, 630090 Russia; Novosibirsk State University, Novosibirsk, 630090 Russia, e-mail: volkov@math.nsc.ru.

Cite this article as: Yu.S.Volkov. Convergence of quartic interpolation splines, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2019, vol. 25, no. 2, pp. 67–74.