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POLYHEDRAL COMPLEMENTARITY ON A SIMPLEX. METHOD OF MEETING PATHS FOR DECREASING QUASI-REGULAR MAPPINGS

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The paper explores the mathematical basis of a novel polyhedral complementarity approach proposed by the author for finding an economic equilibrium in a linear exchange model and its variations. The equilibrium problem reduces to finding fixed points of point-to-set mappings of the price simplex to itself. As a result, we obtain a polyhedral complementarity problem generated by a pair of polyhedral complexes in duality. The class of quasi-regular mappings, which is a new class of decreasing mappings having no analogs in \mathbb{R}^n , is considered. The procedure of meeting paths, which generalizes the known Lemke method for linear complementarity problems, is studied. It is shown that in the case under consideration the procedure has the property of monotonicity characteristic of linear complementarity problems with positive principal minors of the constraint matrix (P-class). The uniqueness of the desired fixed point is a consequence of monotonicity.

Keywords: simplex, polyhedral complex, complementarity, monotonicity, fixed point, algorithm.

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