TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 25 No. 2 **2019**

MSC: 42C10, 47A58

DOI: 10.21538/0134-4889-2019-25-2-258-272

MEAN-SQUARE APPROXIMATION OF FUNCTIONS OF A COMPLEX VARIABLE BY FOURIER SUMS IN ORTHOGONAL SYSTEMS

M. Sh. Shabozov, M. S. Saidusainov

Assume that $\mathcal{A}(U)$ is the set of functions analytic in the disk $U := \{z : |z| < 1\}$, $L_2^{(r)} := L_2^{(r)}(U)$ for $r \in \mathbb{N}$ is the class of functions $f \in \mathcal{A}(U)$ such that $f^{(r)} \in L_2^{(r)}$, and $W^{(r)}L_2$ is the class of functions $f \in L_2^{(r)}$ satisfying the constraint $\|f^{(r)}\| \le 1$. We find exact values for mean-square approximations of functions $f \in W^{(r)}L_2$ and their successive derivatives $f^{(s)}$ $(1 \le s \le r-1,\ r \ge 2)$ in the metric of the space L_2. A similar problem is solved for the class $W_2^{(r)}(\mathscr{K}_m, \Psi)$ $(r \in \mathbb{Z}_+,\ m \in \mathbb{N})$ of functions $f \in L_2^{(r)}$ such that the $\mathscr{K}$-functional of their rth derivative satisfies the condition

$$\mathscr{K}_m\left(f^{(r)}, t^m\right) \le \Psi(t^m), \quad 0 < t < 1,$$

where Ψ is some increasing majorant and $\Psi(0) = 0$.

Keywords: generalized modulus of continuity, generalized translation operator, orthonormal system, Jackson–Stechkin inequality, $\mathscr{K}$-functional.

REFERENCES

1. Smirnov V.I., Lebedev N., A. *Functions of a complex variable. Constructive theory*. London: Iliffe Books Ltd., 1968, 488 p. Original Russian text published in Smirnov V.I., Lebedev N.A. *Konstruktivnaya teoriya funktsii kompleksnogo peremennogo*. Moscow; Leningrad: Nauka Publ., 1964, 440 p.
2. Abilov V.A., Abilova F.V., Kerimov M.K. Sharp estimates for the convergence rate of Fourier series of complex variable functions in $L_2(D, p(z))$. *Comput. Math. Math. Physics*, 2010, vol. 50, no. 6, pp. 946–950. doi: 10.1134/S0965542510060023 .
3. Shabozov M.Sh., Saidusaynov M.S. Mean-square approximation of complex variable functions by Fourier series in the weighted Bergman space. *Vladikavkaz. Mat. Zh.*, 2018, vol. 20, no. 1, pp. 86–97 (in Russian). doi: 10.23671/VNC.2018.1.11400 .
4. Shabozov M.Sh., Saidusaynov M.S. Upper bounds for the approximation of certain classes of functions of a complex variable by Fourier series in the space L_2 and n-widths. *Math. Notes*, 2018, vol. 103, no. 3-4, pp. 656–668. doi: 10.1134/S0001434618030343 .
5. Shevchuk A.I. *Priblizhenie mnogochlenami i sledy neprerývnykh na otrezke funktsii* [Approximation by polynomials and tracks of continuous functions on the segment]. Kiev: Naukova Dumka Publ., 1992, 255 p.
6. Vakarchuk S.B. Exact constants in Jackson-type inequalities and exact values of widths. *Math. Notes*, 2005, vol. 78, no. 5, pp. 735–739. doi 10.1007/s11006-005-0176-y .
7. Vakarchuk S.B. Jackson-type inequalities and widths of function classes in L_2. *Math. Notes*, 2006, vol. 80, no. 1, pp. 11–18. doi: 10.1007/s11006-006-0102-y .
8. Shabozov M.Sh., Vakarchuk S.B. On the best approximation of periodic functions by trigonometric polynomials and the exact values of widths of function classes in L_2. *Anal. Math.*, 2012, vol. 38, no. 2, pp. 147–159 (in Russian).
9. Shabozov M.Sh., Vakarchuk S.B., Zabutnaya V.I. Structural characteristics of functions from L_2 and the exact values of widths of some functional classes. *J. Math. Sci.*, 2015, vol. 206, no. 1, pp. 97–114. doi: 10.1007/s10958-015-2296-6 .

10. Vakarchuk S.B., Shabozov M.Sh. The widths of classes of analytic functions in a disc. *Sb. Math.*, 2010, vol. 201, no. 8, pp. 1091–1110. doi: 10.1070/SM2010v201n08ABEH004104 .
11. Shabozov M.Sh., Yusupov G.A. Best approximation methods and widths for some classes of functions in $H_{q,\rho}$, $1 \le q \le \infty$, $0 < \rho \le 1$. *Siberian Math. J.*, 2016, vol. 57, no. 2, pp. 369–376. doi: 10.1134/S0037446616020191 .
12. Bergh J., Lofstrom J. *Interpolation spaces. An introduction.* Berlin; Heidelberg; N Y: Springer-Verlag, 1976, 220 p. doi: 10.1007/978-3-642-66451-9 . Translated to Russian under the title *Interpolyatsionnye prostranstva. Vvedenie.* Moscow: Mir Publ., 1980. 264 p.
13. Mhaskar N.H. Weighted polynomial approximation. *J. Approx. Theory*, 1986, vol. 46, no. 1, pp. 100–110. doi: 10.1016/0021-9045(86)90089-4 .
14. Ditzian Z., Totik V. $\mathscr{K}$-functionals and best polynomial approximation in weighted $L^p(\mathbb{R})$. *J. Approx. Theory*, 1986, vol. 46, no. 1, pp. 38–41. doi: 10.1016/0021-9045(86)90084-5 .
15. Vakarchuk S.B. Mean approximation of functions on the real axis by algebraic polynomials with Chebyshev–Hermite weight and widths of function classes. *Math. Notes*, 2014, vol. 95, no. 5, pp. 599–614. doi: 10.1134/S0001434614050046 .
16. Shabozov M.Sh., Tukhliev K. $\mathscr{K}$-functionals and exact values of n-widths of certain classes of functions in $L_2\left((\sqrt{1-x^2})^{-1},[-1,1]\right)$ space. *Izv. Tul. Gos. Univ. Estestv. Nauki*, 2014, vol. 1, no. 1, pp. 83–97.
17. Saidusaynov M.S. $\mathscr{K}$-functionals and exact values of n-widths in the Bergman space. *Ural Math. J.*, 2017, vol. 3, no. 2(5), pp. 74–81. doi: 10.15826/umj.2017.2.010 .
18. Vakarchuk S.B., Vakarchuk M.B. On inequalities of Kolmogorov type for analytic functions in a disk. *Dnipr. Univ. Math. Bull.*, 2012, vol. 17, no. 6/1, pp. 82–88 (in Russian).

Received February 28, 2019
Revised May 24, 2019
Accepted May 27, 2019

Mirgand Shabozovich Shabozov, Dr. Sci. (Phys.-Math.), Prof., Tajik National University, Dushanbe, 734025 Republic of Tajikistan; University of Central Asia, Dushanbe, SPCE, 734013 Republic of Tajikistan, e-mail: shabozov@mail.ru .

Mukim Saidusaynovich Saidusaynov, Cand. Sci. (Phys.-Math.), Tajik National University, Dushanbe, 734025 Republic of Tajikista; University of Central Asia, Dushanbe, SPCE, 734013 Republic of Tajikistan, e-mail: smuqim@gmail.com .

Cite this article as: M. Sh. Shabozov, M. S. Saidusainov. Mean-square approximation of functions of a complex variable by Fourier sums in orthogonal systems, *Trudy Instituta Matematiki i Mekhaniki URO RAN*, 2019, vol. 25, no. 2, pp. 258–272 .