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SUPERCOMPACT SPACES OF ULTRAFILTERS AND MAXIMAL LINKED SYSTEMS

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We consider maximal linked systems and ultrafilters of broadly understood measurable spaces; each of these measurable spaces is defined by a π-system of subsets of a nonempty set with "zero" and "one" (a π-system is a family of sets closed under finite intersections). There are specific types of π-systems: semialgebras and algebras of sets as well as topologies and families of closed sets in topological spaces. The problem of supercompactness of an ultrafilter space equipped by a Wallman type topology is studied, and certain properties of bitopological spaces whose points are maximal linked systems and ultrafilters of the corresponding measurable space are analyzed. We also investigate conditions on a measurable space under which maximal linked systems and ultrafilters can be identified, which makes it possible to equip a set of ultrafilters with a supercompact topology of Wallman type by means of a direct application of a similar construction of the space of maximal linked systems. We also give some variants of measurable spaces with algebras of sets for which the Wallman topology of the ultrafilter space is supercompact, although, in general, there exist maximal linked systems of the corresponding measurable space that are not ultrafilters. This scheme is based on a special construction of homeomorphism for Wallman topologies. We give specific examples of measurable spaces for which the supercompact ultrafilter space is realized.

Keywords: algebra of sets, homeomorphism, maximal linked system, ultrafilter.

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