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**N. P. KUPTSOV'S METHOD FOR THE CONSTRUCTION OF AN EXTREMAL  
FUNCTION IN AN INEQUALITY BETWEEN UNIFORM NORMS  
OF DERIVATIVES OF FUNCTIONS ON THE HALF-LINE**

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On the class  $L_{\infty}^4(\mathbb{R}_+)$  of functions  $f \in C(\mathbb{R}_+)$  having a locally absolutely continuous third-order derivative on the half-line  $\mathbb{R}_+$  and such that  $f^{(4)} \in L_{\infty}(\mathbb{R}_+)$ , we study an extremal function in the exact inequalities

$$\|f^{(j)}\| \leq C_{4,j}(\mathbb{R}_+) \|f\|^{1-j/4} \|f^{(4)}\|^{j/4}, \quad j = \overline{1,3}, \quad f \in L_{\infty}^4(\mathbb{R}_+).$$

We present N. P. Kuptsov's earlier unpublished method for the construction of an extremal function, which is an ideal spline of the fourth degree. The method is iterative; it finds the knots and coefficients of the spline and calculates the values  $C_{4,j}(\mathbb{R}_+)$ . The proposed approach differs from the approach of Schoenberg and Cavaretta (1970) and allows to understand the structure of the problem more deeply.

Keywords: inequality between norms of derivatives of functions, four times differentiable functions, uniform norm, half-line.

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