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APPROXIMATION OF DERIVATIVES OF ANALYTIC FUNCTIONS FROM ONE HARDY CLASS BY ANOTHER HARDY CLASS

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In the Hardy space $\mathcal{H}^p(D_\varrho)$, $1 \le p \le \infty$, of functions analytic in the disk $D_\varrho = \{z \in \mathbb{C} : |z| < \varrho\}$, we denote by $NH^p(D_\varrho)$, $N > 0$, the class of functions whose L^p-norm on the circle $\gamma_\varrho = \{z \in \mathbb{C} : |z| = \varrho\}$ does not exceed the number N and by $\partial H^p(D_\varrho)$ the class consisting of the derivatives of functions from $1H^p(D_\varrho)$. We consider the problem of the best approximation of the class $\partial H^p(D_\rho)$ by the class $NH^p(D_R)$, $N > 0$, with respect to the L^p-norm on the circle γ_r, $0 < r < \rho < R$. The order of the best approximation as $N \to +\infty$ is found:

$$\mathcal{E}\left(\partial H^p(D_\rho), NH^p(D_R)\right)_{L^p(\Gamma_r)} \asymp N^{-\beta/\alpha} \ln^{1/\alpha} N, \quad \alpha = \frac{\ln R - \ln \rho}{\ln R - \ln r}, \quad \beta = 1 - \alpha.$$

In the case where the parameter N belongs to some sequence of intervals, the exact value of the best approximation and a linear method implementing it are obtained. A similar problem is considered for classes of functions analytic in rings.

Keywords: analytic functions, Hardy class, best approximation of a class by a class.

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