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A NUMERICAL METHOD FOR THE SOLUTION OF BOUNDARY VALUE PROBLEMS FOR A HOMOGENEOUS EQUATION WITH THE SQUARED LAPLACE OPERATOR WITH THE USE OF INTERPOLATION WAVELETS

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We present an effective numerical method for the recovery of biharmonic functions in a disk from continuous boundary values of these functions and of their normal derivatives using wavelets that are harmonic in the disk and interpolating on its boundary on dyadic rational grids. The expansions of solutions of boundary value problems into cumbersome interpolation series in the wavelet basis are folded into sequences of their partial sums that are compactly presentable in the subspace bases of the corresponding multiresolution analysis (MRA) of Hardy spaces $h_\infty(K)$ of functions harmonic in the disk. Effective estimates are obtained for the approximation of solutions by partial sums of any order in terms of the best approximation of the boundary functions by trigonometric polynomials of a slightly smaller order. As a result, to provide the required accuracy of the representation of the unknown biharmonic functions, one can choose in advance the scaling parameter of the corresponding MRA subspace such that the interpolation projection to this space defines a simple analytic representation of the corresponding partial sums of interpolation series in terms of appropriate compressions and shifts of the scaling functions, skipping complicated iterative procedures for the numerical construction of the coefficients of expansion of the boundary functions into series in interpolation wavelets. We write solutions using interpolation and interpolation-orthogonal wavelets based on modified Meyer wavelets, the last are convenient to apply if the boundary values of the boundary value problem are given approximately, for example, are found experimentally. In this case, one can employ the usual, well-known procedures of discrete orthogonal wavelet transformations for the analysis and refinement (correction) of the boundary values.

Keywords: biharmonic function, boundary value problems, interpolation wavelets, multiresolution analysis (MRA).

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