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**ON A LIST  $(k, l)$ -COLORING OF INCIDENTORS IN MULTIGRAPHS  
OF EVEN DEGREE FOR SOME VALUES OF  $k$  AND  $l$**

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The problem of a list  $(k, l)$ -coloring of incidentors of a directed multigraph without loops is studied in the case where the lists of admissible colors for incidentors of each arc are integer intervals. According to a known conjecture, if the lengths of these interval are at least  $2\Delta + 2k - l - 1$  for every arc, where  $\Delta$  is the maximum degree of the multigraph, then there exists a list  $(k, l)$ -coloring of incidentors. We prove this conjecture for multigraphs of even maximum degree  $\Delta$  with the following parameters:

- $l \geq k + \Delta/2$ ;
- $l < k + \Delta/2$  and  $k$  or  $l$  is odd;
- $l < k + \Delta/2$  and  $k = 0$  or  $l - k = 2$ .

Keywords: list coloring, incidentors,  $(k, l)$ -coloring.

**REFERENCES**

1. Vasil'eva E.I., Pyatkin A.V. On list incidentor  $(k, l)$ -coloring. *J. Appl. Industrial Math.*, 2017, vol. 11, no. 1, pp. 125–129. doi: 10.1134/S1990478917010148.
2. Vizing V.G. Coloring the graph vertices with some prescribed colors. In: *Methods of Discrete Analysis in the Theory of Codes and Schemes*, vol. 29 (Inst. Math. SO AN SSSR, Novosibirsk, 1976), pp. 3–10 (in Russian).
3. Vizing V.G. Incidentor coloring of multigraphs in prescribed colors. *Diskretn. Anal. Issled. Oper. Ser. 1*, 2000, vol. 7, no. 1, pp. 32–39 (in Russian).
4. Vizing V.G., Melnikov L.S., Pyatkin A.V. On  $(k, l)$ -coloring of incidentors. *Diskretn. Anal. Issled. Oper. Ser. 1*, 2000, vol. 7, no. 4, pp. 29–37 (in Russian).
5. Pyatkin A.V. Some optimization problems of scheduling the transmission of messages in a local communication network. In: A.D. Korshunov (ed.), *Operations Research and Discrete Analysis*. Netherlands: Kluwer Acad. Publ., 1997, pp. 227–232. doi: 10.1007/978-94-011-5678-3\_17.
6. Bollobas B., Harris A.J. List-colorings of graphs. *Graphs Combin.*, 1985, vol. 1, no. 1, pp. 115–127. doi: 10.1007/BF02582936 .
7. Borodin O.V., Kostocka A.V., Woodall D.R. List edge and list total colorings of multigraphs. *J. Combin. Theory, Ser. B*, 1997, vol. 71, no. 2, pp. 184–204. doi: 10.1006/jctb.1997.1780 .
8. Diestel R. *Graph theory*. Graduate Texts in Math., vol. 173, 5th ed. Heidelberg: Springer-Verlag, 2016, 448 p. ISBN: 978-3-662-53621-6 .
9. Erdős P., Rubin A.L., Taylor H. Choosability in graphs. In: *Proc. West Coast Conf. on Combinatorics, Graph Theory and Computing*, Vol 26 of Congressus Numerantium, Arcata, California, 1979, pp. 125–157.
10. Häggkvist R., Chetwynd A.G. Some upper bounds on the total and list chromatic numbers of multigraphs. *J. Graph Theory*, 1992, vol. 16, no. 2, pp. 503–516. doi: 10.1002/jgt.3190160206 .
11. Hall P. On representatives of subsets. *J. London Math. Soc.*, 1935, vol. 10, no. 1, pp. 26–30. doi: 10.1112/jlms/s1-10.37.26 .
12. West D.B. *Introduction to graph theory*. New Jersey: Prentice Hall, 2001, 588 p. ISBN: 0-13-014400-2 .

13. Woodall D.R. List colourings of graphs. In: J. W. P. Hirschfeld (ed.) *Surveys in combinatorics*, 2001, London Math. Soc. Lecture Note Ser., vol. 288, Cambridge: Cambridge Univ. Press, 2001, pp. 269–301. ISBN: 0-521-00270-2 ,

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