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APPROXIMATION OF FUNCTIONS BY n-SEPARATE WAVELETS IN THE SPACES $L^p(\mathbb{R})$, $1 \le p \le \infty$

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We consider the orthonormal bases of n-separate MRAs and wavelets constructed by the author earlier. The classical wavelet basis of the space $L^2(\mathbb{R})$ is formed by shifts and compressions of a single function ψ. In contrast to the classical case, we consider a basis of $L^2(\mathbb{R})$ formed by shifts and compressions of n functions ψ^s, $s = 1, \dots, n$. The constructed n-separate wavelets form an orthonormal basis of $L^2(\mathbb{R})$. In this case, the series $\sum_{s=1}^{n} \sum_{j\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} \langle f, \psi^s_{nj+s}\rangle \psi^s_{nj+s}$ converges to the function f in the space $L^2(\mathbb{R})$. We write additional constraints on the functions φ^s and ψ^s, $s = 1, \dots, n$, that provide the convergence of the series to the function f in the spaces $L^p(\mathbb{R})$, $1 \le p \le \infty$, in the norm and almost everywhere.

Keywords: wavelet, scaling function, basis, multiresolution analysis.

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