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MARKOV'S WEAK INEQUALITY FOR ALGEBRAIC POLYNOMIALS ON A CLOSED INTERVAL

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For a real algebraic polynomial P_n of degree n, we consider the ratio $M_n(P_n)$ of the measure of the set of points from $[-1,1]$ where the absolute value of the derivative exceeds n^2 to the measure of the set of points where the absolute value of the polynomial exceeds 1. We study the supremum $M_n = \sup M_n(P_n)$ over the set of polynomials P_n whose uniform norm on $[-1,1]$ is greater than 1. It is known that M_n is the supremum of the exact constants in Markov's inequality in the class of integral functionals generated by a nondecreasing nonnegative function. In this paper we prove the estimates $1 + 3/(n^2-1) \le M_n \le 6n+1$ for $n \ge 2$.

Keywords: Markov's inequality, algebraic polynomials, Lebesgue measure, weak-type inequalities.

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