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ON THE COINCIDENCE OF REPRODUCING KERNEL HILBERT SPACES CONNECTED BY A SPECIAL TRANSFORMATION

V. V. Napalkov, V. V. Napalkov, Jr

We consider two reproducing kernel Hilbert spaces H_1 and H_2 consisting of complex-valued functions given on some sets $\Omega_1 \subset \mathbb{C}^n$ and $\Omega_2 \subset \mathbb{C}^m$, respectively. The norms in H_1 and H_2 have integral form:

$$\|f\|^2_{H_1} = \int_{\Omega_1} |f(z)|^2 \, d\mu(z), \quad f \in H_1; \qquad \|q\|^2_{H_2} = \int_{\Omega_2} |q(t)|^2 \, d\nu(t), \quad q \in H_2.$$

Let $\{E(\cdot, z)\}_{z\in\Omega_2}$ be some complete system of functions in the space H_1. Define

$$\widetilde{f}(z) \stackrel{\text{def}}{=} (E(\cdot, z), f)_{H_1} \ \forall z \in \Omega_2, \quad \widetilde{H}_1 = \{\widetilde{f},\, f \in H_1\}, (\widetilde{f}_1, \widetilde{f}_2)_{\widetilde{H}_1} \stackrel{\text{def}}{=} (f_2, f_1)_{H_1}, \|\widetilde{f}_1\|_{\widetilde{H}_1} = \|f_1\|_{H_1} \ \ \forall \widetilde{f}_1, \widetilde{f}_2 \in \widetilde{H}_1.$$

We study the question of coincidence of the spaces $\widetilde{H}_1$ and H_2, i.e., the conditions under which these spaces consist of the same functions and have equal norms. The following criterion of coincidence is obtained: $\widetilde{H}_1 = H_2$ if and only if there exists a linear continuous one-to-one unitary operator $\mathcal{A}$ from $\overline{H}_1$ onto H_2 that for any $\xi \in \Omega_1$ takes the function $K_{\overline{H}_1}(\cdot, \xi)$ to the function $E(\xi, \cdot)$. Here $\overline{H}_1$ is the space consisting of the complex conjugates of functions from H_1 and $K_{\overline{H}_1}(t, \xi)$, $t, \xi \in \Omega_1$, is the reproducing kernel of the space $\overline{H}_1$. We also obtain some equivalent statements and a criterion for the coincidence of H_1 and H_2.

Keywords: Bargmann–Fock space, operator of multiplication by a function, expansion systems similar to orthogonal systems, reproducing kernel Hilbert space.

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Valentin Vasilievich Napalkov Dr. Phys.-Math. Sci., Prof., Corresponding Member of RAS, Institute of Mathematics, Ufa Federal Research Centre of the Russian Academy of Sciences, Ufa, 450077 Russia, e-mail: napalkov@matem.anrb.ru .

Valerii Valentinovich Napalkov, Cand. Sci. (Phys.-Math.), Institute of Mathematics, Ufa Federal Research Centre of the Russian Academy of Sciences, Ufa, 450077 Russia, e-mail: vnap@mail.ru .

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