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## CONSTRUCTION OF OPTIMAL COVERS BY DISKS OF DIFFERENT RADII FOR CONVEX PLANAR SETS

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We consider the problem of constructing an optimal cover of a planar set  $M$  by the union of a given number of disks. In the general case, the radii of the disks are assumed to be different; each radius is the product of some positive factor specific for each disk and a parameter  $r$ , which is common for all elements of the cover. The optimality criterion is the minimum of  $r$  under the condition that  $M$  is a subset of the union of the disks. For a set of points  $S$ , we write the value of  $r$  that defines the minimum radius of the disks centered at the points of  $S$  and implementing a cover of  $M$ . Expressions are found that analytically describe the impact zones (the so-called generalized Dirichlet zones) of the points of  $S$ , which differ significantly from the expressions for the case of congruent circles. A procedure for the iterative correction of coordinates of  $S$  based on finding Chebyshev centers of impact zones of points is proposed. It is shown that the procedure does not degrade the properties of the cover, while its parameters can be changed in the process of starting the software complex. Numerical experiments on the construction of optimal covers by families of disks were carried out with different coefficients defining the radii of the disks. Various convex polygons were taken as the set  $M$ , and the results were visualized.

Keywords: optimal cover, generalized Dirichlet zone, Chebyshev center, iterative algorithm, minimization.

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