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ANALYTIC EMBEDDING OF THREE-DIMENSIONAL SIMPLICIAL GEOMETRIES

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The study of maximum mobility geometries is of great importance for modern mathematics. The maximum mobility of an n-dimensional geometry defined by a function f of a pair of points means the existence of an $n(n+1)/2$-dimensional transformation group fixing this function. There are a number of known maximum mobility geometries (Euclidean, symplectic, Lobachevskian, etc.), but there is no complete classification of such geometries. In this paper, we solve one of such classification problems by the embedding method. The essence of the method is as follows: from the function g of a pair of points of a three-dimensional geometry, we find all nondegenerate functions f of a pair of points of four-dimensional geometries that are invariants of the Lie group of transformations of dimension 10. In this paper, g are nondegenerate functions of a pair of points of three simplicial three-dimensional geometries:

$$g = \frac{y_i - y_j}{x_i - x_j} + 2z_i + 2z_j, \quad g = \ln\frac{y_i - y_j}{x_i - x_j} + 2z_i + 2z_j, \quad g = \arctan\frac{y_i - y_j}{x_i - x_j} + 2z_i + 2z_j.$$

These geometries are locally maximally mobile, which means that their groups of motions are six-dimensional. The problem solved in this paper is reduced to the solution of special functional equations by analytical methods. The solutions are sought in the form of Taylor series. The Maple 15 mathematical software package is used for the enumeration of various options. As a result, we obtain only degenerate functions of a pair of points, which do not define a maximum mobility geometry.

Keywords: functional equation, maximum mobility geometry, group of motions, simplicial geometry.

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