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SELF-INTERSECTIONS IN PARAMETRIZED SELF-SIMILAR SETS UNDER TRANSLATIONS AND EXTENSIONS OF COPIES

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We study the problem of pairwise intersections $F_i(K_t) \cap F_j^t(K_t)$ of different copies of a self-similar set K_t generated by a system $\mathcal{F}_t = \{F_1, \dots, F_m\}$ of contracting similarities in $\mathbb{R}^n$, where one mapping F_j^t depends on a real or vector parameter t. Two cases are considered: the parameter $t \in \mathbb{R}^n$ specifies a translation of a mapping $F_j^t(x) = G(x) + t$, and the parameter $t \in (a, b)$ is the similarity coefficient of a mapping $F_j^t(x) = tG(x) + h$, where $0 < a < b < 1$ and G is an isometry of $\mathbb{R}^n$. We impose some constraints on the similarity coefficients of mappings of the system $\mathcal{F}_t$ and require that the similarity dimension of the system does not exceed some number s. For such systems it is proved that the Hausdorff dimension of the set of parameters t for which the intersection $F_i(K_t) \cap F_j^t(K_t)$ is nonempty does not exceed $2s$. The obtained results are applied to the problem of checking the strong separation condition for a system $\mathcal{F}_\tau = \{F_1^\tau, \dots, F_m^\tau\}$ of contraction similarities depending on a parameter vector $\tau = (t_1, \dots, t_m)$. Two cases are considered: τ is a vector of translations of mappings $F_i^\tau(x) = G_i(x) + t_i$, $t_i \in \mathbb{R}^n$, and τ is a vector of similarity coefficients of mappings $F_i^\tau(x) = t_i G_i(x) + h_i$, $t_i \in (a, b)$, where $0 < a < b < 1$ and all G_i are isometries in $\mathbb{R}^n$. In both cases we find sufficient conditions for the system $\mathcal{F}_\tau$ to satisfy the strong separation condition for almost all values of τ. We also consider the easier problem of the intersection $A \cap f_t(B)$ for a pair of compact sets A and B in the space $\mathbb{R}^n$. Two cases are considered: $f_t(B) = B + t$ for $t \in \mathbb{R}^n$, and $f_t(B) = tB$ for $t \in \mathbb{R}$, where the closure of B does not contain the origin. In both cases it is proved that the Hausdorff dimension of the set of parameters t for which the intersection $A \cap f_t(B)$ is nonempty does not exceed $\dim_H(A \times B)$. Consequently, when the dimension of the product $A \times B$ is small enough, the empty intersection $A \cap f_t(B)$ is guaranteed for almost all values of t in both cases.

Keywords: self-similar fractal, general position, strong separation condition, Hausdorff dimension.

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