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INTEGRABILITY PROPERTIES OF FUNCTIONS WITH A GIVEN BEHAVIOR OF DISTRIBUTION FUNCTIONS AND SOME APPLICATIONS

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We establish that if the distribution function of a measurable function v given on a bounded domain Ω of $\mathbb{R}^n$ $(n \geqslant 2)$ satisfies, for sufficiently large k, the estimate $\operatorname{meas}\{|v| > k\} \leqslant k^{-\alpha}\varphi(k)/\psi(k)$, where $\alpha > 0$, $\varphi\colon [1, +\infty) \to \mathbb{R}$ is a nonnegative nonincreasing measurable function such that the integral of the function $s \to \varphi(s)/s$ over $[1, +\infty)$ is finite, and $\psi\colon [0, +\infty) \to \mathbb{R}$ is a positive continuous function with some additional properties, then $|v|^\alpha \psi(|v|) \in L^1(\Omega)$. In so doing, the function ψ can be bounded or unbounded. We give corollaries of the corresponding theorems for some specific ratios of the functions φ and ψ. In particular, we consider the case where the distribution function of a measurable function v satisfies, for sufficiently large k, the estimate $\operatorname{meas}\{|v| > k\} \leqslant Ck^{-\alpha}(\ln k)^{-\beta}$ with $C, \alpha > 0$ and $\beta \geqslant 0$. In this case, we strengthen our previous result for $\beta > 1$ and, on the whole, we show how the integrability properties of the function v differ depending on which of the intervals $[0, 1]$ or $(1, +\infty)$ contains β. We also consider the case where the distribution function of a measurable function v satisfies, for sufficiently large k, the estimate $\operatorname{meas}\{|v| > k\} \leqslant Ck^{-\alpha}(\ln \ln k)^{-\beta}$ with $C, \alpha > 0$ and $\beta \geqslant 0$. We give examples showing the accuracy of the obtained results in the corresponding scales of classes close to $L^\alpha(\Omega)$. Finally, we give applications of these results to entropy and weak solutions of the Dirichlet problem for nonlinear elliptic second-order equations with right-hand side in some classes close to $L^1(\Omega)$ and defined by the logarithmic function or its double composition.

Keywords: integrability, distribution function, nonlinear elliptic equations, right-hand side in classes close to L^1, Dirichlet problem, weak solution, entropy solution.

REFERENCES

1. Talenti G. Elliptic equations and rearrangements. *Ann. Sc. Norm. Super. Pisa. Cl. Sci. (4)*, 1976, vol. 3, no. 4, pp. 697–718.
2. Bénilan Ph., Boccardo L., Gallouët T., Gariepy R., Pierre M., Vazquez J.L. An L^1-theory of existence and uniqueness of solutions of nonlinear elliptic equations. *Ann. Sc. Norm. Super. Pisa. Cl. Sci. (4)*, 1995, vol. 22, no. 2, pp. 241–273.
3. Kovalevskii A.A. Integrability of solutions of nonlinear elliptic equations with right-hand sides from classes close to L^1. *Math. Notes*, 2001, vol. 70, no. 3, pp. 337–346. doi: 10.1023/A:1012343728148 .
4. Kovalevsky A.A. General conditions for limit summability of solutions of nonlinear elliptic equations with L^1-data. *Nonlinear Anal.*, 2006, vol. 64, no. 8, pp. 1885–1895. doi: 10.1016/j.na.2005.08.008 .
5. Boccardo L., Gallouet T. Nonlinear elliptic equations with right hand side measures. *Commun. Partial Diff. Eq.*, 1992, vol. 17, no. 3–4, pp. 641–655. doi: 10.1080/03605309208820857 .
6. Kovalevsky A.A., Skrypnik I.I., Shishkov A.E. *Singular solutions of nonlinear elliptic and parabolic equations*. Berlin: De Gruyter, 2016, De Gruyter Ser. Nonlinear Anal. Appl., vol. 24, 434 p. Original Russian text published in *Singulyarnye resheniya nelineinykh ellipticheskikh i parabolicheskikh uravnenii*, Kiev: Naukova Dumka, 2010, 499 p. doi: 10.1515/9783110332247 .
7. Boccardo L., Gallouet T. $W_0^{1,1}$ solutions in some borderline cases of Calderon–Zygmund theory. *J. Diff. Eq.*, 2012, vol. 253, no. 9, pp. 2698–2714. doi: 10.1016/j.jde.2012.07.003 .
8. Boccardo L., Gallouet T. Summability of the solutions of nonlinear elliptic equations with right hand side measures. *J. Convex Anal.*, 1996, vol. 3, no. 2, pp. 361–365.
9. Kovalevsky A.A. A priori properties of solutions of nonlinear equations with degenerate coercivity and L^1-data. *J. Math. Sci.*, 2008, vol. 149, no. 5, pp. 1517–1538. doi: 10.1007/s10958-008-0080-6 .

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