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## ON THE SOLVABILITY OF A FINITE GROUP WITH SEMINORMAL OR SUBNORMAL SCHMIDT SUBGROUPS OF ONE OF ITS MAXIMAL SUBGROUPS

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A Schmidt group is a finite non-nilpotent group all of whose proper subgroups are nilpotent. A group with a nilpotent maximal subgroup is known to be solvable if the derived subgroup of a Sylow 2-subgroup of a maximal subgroup is contained in the center of the Sylow 2-subgroup. If a maximal subgroup of a group is non-nilpotent, then it has a Schmidt subgroup. The structure of a group and, in particular, its solvability, depend on the properties of Schmidt subgroups of its maximal subgroup. In this paper, we establish the solvability of a finite group such that some Schmidt subgroups of its maximal subgroup are seminormal or subnormal in the group.

Keywords: finite group, solvable group, Schmidt subgroup, subnormal subgroup, seminormal subgroup, maximal subgroup.

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