TRUDY INSTITUTA MATEMATIKI I MEKHANIKI UrO RAN

Vol. 25 No. 1   **2019**

MSC: 47H07, 54C60, 34K09, 55M20
DOI: 10.21538/0134-4889-2019-25-1-45-54

ON THE EXISTENCE AND ESTIMATES OF SOLUTIONS
TO FUNCTIONAL EQUATIONS

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We consider the issues of solvability of operator inclusions in partially ordered spaces. We use the notion of ordered covering of multivalued mappings proposed by A. V. Arutyunov, E. S. Zhukovskiy, and S. E. Zhukovskiy in their paper "Coincidence points principle for set-valued mappings in partially ordered spaces," Topology Appl. **201**, 330–343 (2016). A statement on the preservation of properties of an ordered covering under antitone perturbations is proved. Conditions for an ordered covering of the multivalued Nemytskii operator acting from the space of essentially bounded functions to the space of measurable functions are obtained. More exactly, it is established that, if the multivalued mapping $f(t, x)$ is orderly covering in the second argument (in the space $\mathbb{R}^n$), then the corresponding Nemytskii operator (defined as the set of measurable sections of the mapping $f(t, x(t))$) is also orderly covering. These results are used to study a functional inclusion with a deviating argument of the form $0 \in g(t, x(h(t)), x(t))$. It is assumed that the multivalued mapping $g(t, x, y)$ is nonincreasing in the second argument and is orderly covering in the third argument. For this inclusion, a solution existence theorem is proved and estimates of solutions are obtained.

Keywords: ordered space, multivalued orderly covering mapping, multivalued Nemytskii operator, space of measurable functions, functional inclusion, existence of a solution.

REFERENCES

1. Arutyunov A.V. Covering mappings in metric spaces, and fixed points. *Dokl. Math.*, 2007, vol. 76, no. 2, pp. 665–668. doi: 10.1134/S1064562407050079 .

2. Avakov E.R., Arutyunov A.V., Zhukovskii E.S. Covering mappings and their applications to differential equations unsolved for the derivative. *Differ. Equ.*, 2009, vol. 45, no. 5, pp. 627–649. doi: 10.1134/S0012266109050024 .

3. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E. Covering mappings and well-posedness of nonlinear Volterra equations. *Nonlinear Analysis: Theory, Methods and Applications*, 2012, vol. 75, no. 3, pp. 1026–1044. doi: 10.1016/j.na.2011.03.038 .

4. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E. Coincidence points principle for mappings in partially ordered spaces. *Topology Appl.*, 2015, vol. 179, pp. 13–33. doi: 10.1016/j.topol.2014.08.013 .

5. Arutyunov A.V., Zhukovskiy E.S., Zhukovskiy S.E. Coincidence points principle for set-valued mappings in partially ordered spaces. *Topology Appl.*, 2016, vol. 201, pp. 330–343. doi: 10.1016/j.topol.2015.12.044 .

6. Zhukovskiy E.S. On ordered-covering mappings and implicit differential inequalities. *Differential Equations*, 2016, vol. 52, no. 12, pp. 1539–1556. doi: 10.1134/S0012266116120028 .

7. Zhukovskiy E.S. On order covering maps in ordered spaces and Chaplygin-type inequalities. *St. Petersburg Math. J.*, 2019, vol. 30, no. 1, pp. 73–94. doi: 10.1090/spmj/1530 .

8. Zhukovskiy E.S., Pluzhnikova E.A., Yakubovskaya E.M. On stability of ordered covering of multi-valued mappings under antitone perturbations. *Vestnik Tambovskogo Universiteta. Seriya Estestvennye i Tekhnicheskie Nauki*, 2016, vol. 21, no. 6, pp. 1969–1973 (in Russian). doi: 10.20310/1810-0198-2016-21-6-1969-1973 .

9. Yakubovskaya E.M. About functional inclusions in ordered spaces. *Vestnik Tambovskogo Universiteta. Seriya Estestvennye i Tekhnicheskie Nauki*, 2017, vol. 22, no. 3, pp. 611–614 (in Russian). doi: 10.20310/1810-0198-2017-22-3-611-614 .

10. Aram Arutyunov, Valeriano Antunes de Oliveira, Fernando Lobo Pereira, Evgeniy Zhukovskiy and Sergey Zhukovskiy. On the solvability of implicit differential inclusions. *Applicable Analysis*, 2015, vol. 94, no. 1, pp. 129–143. doi: 10.1080/00036811.2014.891732 .

11. Dmitruk A.V., Milyutin A.A., Osmolovskii N.P. Lyusternik's theorem and the theory of extrema. *Russian Math. Surveys*, 1980, vol. 35, no. 6, pp. 11–51. doi: 10.1070/RM1980v035n06ABEH001973 .

12. Dunford N., Schwartz J.T. *Linear operators. I. General theory.* N Y, Interscience Publishers, 1958, 858 p. ISBN: 0470226056 . Translated to Russian under the title *Lineinye operatory. Obshchaya teoriya.* Moscow: Inostr. Lit. Publ., 1962, 896 p. ISBN: 5-354-00601-5 .

13. Borisovich Yu.G., Gel'man B.D., Myshkis A.D., Obukhovskii V.V. *Vvedenie v teoriyu mnogoznachnykh otobrazhenii i differentsial'nykh vklyuchenii* [Introduction to the theory of multi-valued mappings and differential inclusions]. Moscow: LIBROKOM Publ., 2011, 224 p. ISBN: 978-5-397-01526-4 .

14. Zhukovskiy E.S., Panasenko E.A. Definition of the metric on the space $\text{clos}_\varnothing(X)$ of closed subsets of a metric space X and properties of mappings with values in $\text{clos}_\varnothing(\mathbb{R}^n)$. *Sb. Math.*, 2014, vol. 205, no. 9, pp. 1279–1309. doi: 10.1070/SM2014v205n09ABEH004418 .

15. Vilenkin N.Ya., Krein S.G., et al. *Functional analysis.* Wolters-Noordhoff Series of Monographs and Textbooks on Pure and Applied Mathematics. Groningen, Wolters-Noordhoff Publ., 1972, 379 p. ISBN: 9001909809 . Original Russian text published in Krein S.G. (ed.) *Funkzional'nyi analiz. SMB.* Moscow, Nauka Publ., 1972, 544 p.

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