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A CONTROL PROBLEM FOR A ROD HEATING PROCESS WITH UNKNOWN TEMPERATURE AT THE RIGHT END AND UNKNOWN DENSITY OF THE HEAT SOURCE

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A control problem is considered for the process of heating a rod by varying the temperature at its left end. The exact values of the temperature at the right end of the rod and the heat density function are unknown; only the ranges of their possible values are given. The aim of the control is to ensure that the average temperature of the rod at a fixed time belongs to a given interval. We find necessary and sufficient conditions on the initial temperature of the rod under which the aim of the control can be achieved for any admissible unknown functions. The corresponding heating control at the left end of the rod is constructed.

Keywords: heat equation, temperature, control.

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