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A FELLER TRANSITION KERNEL WITH MEASURE SUPPORTS GIVEN BY A SET-VALUED MAPPING

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Assume that X is a topological space and Y is a separable metric space. Let these spaces be equipped with Borel σ-algebras $\mathcal{B}_X$ and $\mathcal{B}_Y$, respectively. Suppose that $P(x,B)$ is a stochastic transition kernel; i.e., the mapping $x \mapsto P(x,B)$ is measurable for all $B \in \mathcal{B}_Y$ and the mapping $B \mapsto P(x,B)$ is a probability measure for any $x \in X$. Denote by $\mathrm{supp}(P(x,\cdot))$ the topological support of the measure $B \mapsto P(x,B)$. If the transition kernel $P(x,B)$ satisfies the Feller property, i.e., the mapping $x \mapsto P(x,\cdot)$ is continuous in the weak topology on the space of probability measures, then the set-valued mapping $x \mapsto \mathrm{supp}(P(x,\cdot))$ is lower semicontinuous. Conversely, consider a set-valued mapping $x \mapsto S(x)$, where $x \in X$ and $S(x)$ is a nonempty closed subset of a Polish space Y. If $x \mapsto S(x)$ is lower semicontinuous, then, under some general assumptions on the space X, there exists a Feller transition kernel such that $\mathrm{supp}(P(x,\cdot)) = S(x)$ for all $x \in X$.

Keywords: Feller property, transition kernel, topological support of a measure, lower semicontinuous set-valued mapping, continuous branch (selection).

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