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ON THE CONTINUOUS DEPENDENCE OF TRAJECTORIES OF A DIFFERENTIAL INCLUSION ON INITIAL APPROXIMATIONS

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We consider a differential inclusion with an unbounded right-hand side F in the case when this right-hand side satisfies conditions of measurable pseudo-Lipschitzness in a neighborhood of some fixed trajectory $\widehat{x}(\cdot)$. In the space of absolutely continuous functions, we prove a theorem on the existence of a continuous mapping from a certain set of pseudo-trajectories defined in a neighborhood of the trajectory $\widehat{x}(\cdot)$ to a set of trajectories of the differential inclusion with estimates determined by the set of pseudo-trajectories. For the given multivalued mapping F and trajectory $\widehat{x}(\cdot)$, a variational differential inclusion is defined such that the graph of its right-hand side is the lower tangent cone to the graph of the right-hand side F at points of the graph of the trajectory $\widehat{x}(\cdot)$. The existence of a continuous mapping from the set of trajectories of the variational differential inclusion to the set of trajectories of the original differential inclusion is proved with estimates. These properties are an important part of the direct method of deriving necessary optimality conditions in problems with constraints in the form of a differential inclusion.

Keywords: multivalued mapping, differential inclusion, derivative of a multivalued mapping, tangent cone, conditions of measurable pseudo-Lipschitzness of a multivalued mapping, necessary optimality conditions.

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