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**CALCULATION OF ELEMENTS OF A GUIDING PROGRAM PACKAGE
FOR SINGULAR CLUSTERS OF THE SET OF INITIAL STATES
IN THE PACKAGE GUIDANCE PROBLEM**

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A fixed-time package guidance problem is considered for a linear controlled dynamical system with a finite set of initial states. The control set is convex and compact and the target set is convex and closed. The paper focuses on the case where the set of initial states has singular clusters for which the existing algorithm for estimating the elements of a guiding program package is not applicable. It is suggested to consider a perturbed problem of augmented program guidance with a smoothed control set. It is proved that the motions of the original and perturbed problems are close to each other at the terminal time; the corresponding estimates are provided. In the case of an augmented target set with nonempty interior, it is also shown that a solution of the augmented program guidance problem that is precisely guiding to the target set can be obtained by applying the existing algorithm for the perturbed problem with compressed target set. The suggested theoretical constructions are illustrated with a model example.

Keywords: incomplete information, linear dynamical system, guidance problem, program package, singular cluster, smooth approximation.

REFERENCES

1. Osipov Yu.S. Control packages: an approach to solution of positional control problems with incomplete information. *Russian Math. Surveys*, 2006, vol. 61, no. 4, pp. 611–661. doi: 10.4213/rm1760.
2. Kryazhimskiy A.V., Osipov, Y.S. Idealized program packages and problems of positional control with incomplete information. *Proc. Steklov Inst. Math.*, 2010, vol. 268, suppl. 1, pp. 155–174. doi: 10.1134/S0081543810050123.
3. Krasovskii N.N., Subbotin A.I. *Game-theoretical control problems*. N Y: Springer, 1987. 517 p. This book is substantially revised version of the monograph *Pozitsionnye differentsial'nye igry*, Moscow, Nauka Publ., 1974, 456 p.
4. Kryazhimskiy A.V., Osipov Yu.S. On the solvability of problems of guaranteeing control for partially observable linear dynamical systems. *Proc. Steklov Inst. Math.*, 2012, vol. 277, pp. 144–159. doi: 10.1134/S0081543812040104.
5. Kryazhimskii A.V., Strelkovskii N.V. An Open-loop criterion for the solvability of a closed-loop guidance problem with incomplete information: Linear control systems. *Proc. Steklov Inst. Math.*, 2015, vol. 291, suppl. 1, pp. 113–127. doi: 10.1134/S0081543815090084.
6. Strelkovskii N.V. Constructing a strategy for the guaranteed positioning guidance of a linear controlled system with incomplete data. *Moscow Univ. Comput. Math. Cybern.*, 2015, vol. 39, no. 3, pp. 126–134. doi: 10.3103/S0278641915030085.
7. Surkov P.G. The problem of package guidance with incomplete information for a linear control system with a delay. *Comput Math Model*, 2017, vol. 28, no. 4, pp. 504–516. doi: 10.1007/s10598-017-9377-y.
8. Maksimov V.I., Surkov P.G. On the solvability of the problem of guaranteed package guidance to a system of target sets. *Vestn. Udmurtsk. Univ. Mat. Mekh. Komp. Nauki*, 2017, vol. 27, no. 3, pp. 344–354 (in Russian). doi: 10.20537/vm170305.
9. Surkov P.G. The problem of package guidance under incomplete information and integral signal of observation. *Sib. Elektron. Mat. Izv.*, 2018, vol. 15, pp. 373–388 (in Russian). doi: 10.17377/semi.2018.15.034.

10. Strelkovskii N.V., Orlov S.M. Algorithm for constructing a guaranteeing program package in a control problem with incomplete information. *Moscow Univ. Comput. Math. Cybern.*, 2018, vol. 42, no. 2, pp. 69–79. doi: 10.3103/S0278641918020061.
11. Strelkovskii N.V., Orlov S.M. A method for calculation of program package elements for singular clusters. *Proc. Int. Conf. "Systems Analysis: Modeling and Control" in memory of Academician A.V. Kryazhimskiy (Moscow, Russia, 2018)*, pp. 97–99. doi: 10.4213/proc20606.
12. Gindes V.B. Singular control in optimal systems. *Izv. Vyssh. Uchebn. Zaved. Mat.*, 1967, no. 7, pp. 34–42 (in Russian).
13. Avvakumov S.N. Smooth approximation of convex compact sets. *Trudy Inst. Mat. i Mekh. UrO RAN*, 1996, vol. 4, pp. 184–200 (in Russian).
14. Kiselev Yu.N., Avvakumov S.N., Orlov M.V. *Optimal'noe upravlenie. Lineinaya teoriya i prilozheniya*. [Optimal control. Linear theory and applications]. Moscow: MAKS Press, 2007, 272 p. ISBN: 5-89407-288-3.
15. Pontryagin L.S. *Linear differential games. Proc. Steklov Inst. Math.*, 1990, vol. 185, pp. 225–232.
16. Avvakumov S.N., Kiselev Yu.N., Orlov M.V., Methods of solving optimal control problems based on the Pontryagin maximum principle. *Proc. Steklov Inst. Math.*, 1995, vol. 211, pp. 1–27.
17. Avvakumov S.N., Kiselev Yu.N. Support functions of some special sets, constructive smoothing procedures, and geometric difference. In: Osipov Yu.S. (ed.) et al., *Problems in dynamic control. No. 1*. Moscow: Mosk. Gos. Univ., Fak. Vych. Mat. Kib. Publ., 2005, pp. 24–110. ISBN: 5-89407-241-7.

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