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ASYMPTOTICS OF THE SPECTRUM OF A PERIODIC BOUNDARY VALUE PROBLEM FOR A DIFFERENTIAL OPERATOR WITH A SUMMABLE POTENTIAL

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The spectrum of a differential operator of a high odd order with summable potential is studied. The boundary conditions are periodic. The differential equation that defines the differential operator is reduced to the Volterra integral equation. Solving this equation by Picard's method of successive approximations, we find the asymptotics of the fundamental system of solutions of the original differential equation. This fundamental system of solutions is used for the study of periodic boundary conditions. As a result, an equation for the eigenvalues of the differential operator is derived. This equation is a determinant of high order, which is an entire function of the spectral parameter. The indicator diagram corresponding to this function is investigated. The indicator diagram is a regular polygon and determines the location of the eigenvalues of the operator under consideration. As a result, the asymptotic behavior of the eigenvalues of the operator is found in each of the sectors of the complex plane determined by the indicator diagram (of 15th order).

Keywords: spectral parameter, differential operator, summable potential, periodic boundary conditions, asymptotics of solutions of a differential equation, asymptotics of the spectrum.

REFERENCES

1. Levitan B.M., Sargsyan I.S. *Introduction to spectral theory: Selfadjoint ordinary differential operators*. Translations of Mathematical Monographs, vol. 39, Providence: AMS, 1975, 539 p. ISBN: 082181589X . Original Russian text published in Levitan B.M., Sargsyan I.S. *Vvedenie v spektral'nuyu teoriyu*. Moscow: Nauka Publ., 1970, 672 p.
2. Marchenko V.A. Some questions of the theory of one-dimensional linear differential operators of the second order. I. *Trudy Moskov. Mat. Obshch.*, 1952, pp. 327–420 (in Russian).
3. Lidskii V.B., Sadovnichii V.A. Asymptotic formulas for the zeros of a class of entire functions. *Math. USSR-Sb.*, 1968, vol. 4, no. 4, pp. 519–527. doi: 10.1070/SM1968v004n04ABEH002812 .
4. Lidskii V.B., Sadovnichii V.A. Regularized sums of zeros of a class of entire functions. *Funct. Anal. Appl.*, 1967, vol. 1, no. 2, pp. 133–139. doi: 10.1007/BF01076085 .
5. Sadovnichii V.A. The trace of ordinary differential operators of high order. *Math. USSR-Sb.*, 1967, vol. 1, no. 2, pp. 263–288. doi: 10.1070/SM1967v001n02ABEH001979 .
6. Babadzhanov B.A., Khasanov A.B., Yakhshimuratov A.B. On the inverse problem for a quadratic pencil of Sturm–Liouville operators with periodic potential. *Differ. Equ.*, 2005, vol. 41, no. 3, pp. 310–318. doi: 10.1007/s10625-005-0163-9 .
7. Fedotov A.A., Shchetka E.V. Complex WKB method for a difference Schrödinger equation with the potential being a trigonometric polynomial. *St. Petersburg Math. J.*, 2018, vol. 29, no. 2, pp. 363–381. doi: 10.1090/spmj/1497 .
8. Mitrokhin S.I. Formulas for the regularized traces of the second order differential operators with discontinuous coefficients. *Mosc. Univ. Math. Bull.*, 1986, vol. 41, no. 6, pp. 1–5.
9. Mitrokhin S.I. On spectral properties of differential operators with discontinuous coefficients. *Differ. Uravn.*, 1992, vol. 28, no. 3, pp. 530–532 (in Russian).
10. Mitrokhin S.I. On some spectral properties of differential operators of second order with discontinuous positive weight function. *Dokl. Akad. Nauk*, 1997, vol. 356, no. 1, pp. 13–15.
11. Abdullaev A.R. Skachkova E.A. Periodic boundary-value problem for a fourth-order differential equation. *Russian Math. (Iz. VUZ)*, 2013, vol. 57, no. 12, pp. 1–7. doi: 10.3103/S1066369X13120013 .

12. Badanin A.V., Korotyaev E.L. Spectral estimates for a periodic fourth-order operator. *St. Petersburg Math. J.*, 2011, vol. 22, no. 5, pp. 703–736. doi: 10.1090/S1061-0022-2011-01164-1 .
13. Polyakov D.M. Spectral analysis of a fourth order differential operator with periodic and antiperiodic boundary conditions. *St. Petersburg Math. J.*, 2016, vol. 27, no. 5, pp. 789–811. doi: 10.1090/spmj/1417 .
14. Polyakov D.M. On spectral properties of fourth order differential operator with periodic and semiperiodic boundary conditions. *Russian Math. (Iz. VUZ)*, 2015, vol. 59, no. 5, pp. 64–68. doi: 10.3103/S1066369X15050096 .
15. Vinokurov V.A., Sadovnichii V.A. Asymptotics of any order for the eigenvalues and eigenfunctions of the Sturm-Liouville boundary-value problem on a segment with a summable potential. *Izv. Math.*, 2000, vol. 64, no. 4, pp. 695–754. doi: 10.1070/IM2000v064n04ABEH000295 .
16. Mitrokhin S.I. Spectral properties of a fourth-order differential operator with integrable coefficients. *Proc. Steklov Inst. Math.*, 2010, vol. 270, no. 1, pp. 184–193. doi: 10.1134/S0081543810030144 .
17. Mitrokhin S.I. On the spectral properties of odd-order differential operators with integrable potential. *Differ. Equ.*, 2011, vol. 47, no. 12, pp. 1833–1836. doi: 10.1134/S0012266111120123 .
18. Mitrokhin S.I. On spectral properties of the family of high even order differential operators with a summable potential. *Vestnik Voronezh. Gos. Univ. Ser. Matematika*, 2016, no. 4, pp. 121–135 (in Russian).
19. Mitrokhin S.I. On spectral properties of a differential operator with summable coefficients with a retarded argument. *Ufimsk. Mat. Zh.*, 2011, vol. 3, no. 4, pp. 95–115 (in Russian).
20. Mitrokhin S.I. A periodic boundary value problem for a fourth order differential operator with a summable potential. *Vladikavkaz. Mat. Zh.*, 2017, vol. 19, no. 4, pp. 35–49 (in Russian).
21. Naimark M.A. *Linear differential operators*. Mineola, N.Y.: Dover Publ., 2009, 528 p. ISBN: 9780486468976 . Original Russian text published in Naimark M.A., *Lineinye differentsial'nye operatory*, Moscow: Nauka Publ., 1969, 528 p.
22. Bellman R., Cooke K.L. *Differential-difference equations*. N Y: Acad. Press, 1963, 462 p. ISBN: 9780080955148 . Translated to Russian under the title *Differentsial'no-raznostnye uravneniya*. Moscow: Mir Publ., 1967, 548 p.

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