ANALYTIC CONTINUATION METHODS FOR MULTIVALUED FUNCTIONS OF ONE VARIABLE AND THEIR APPLICATION TO THE SOLUTION OF ALGEBRAIC EQUATIONS

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The paper discusses several methods of analytic continuation of a multivalued function of one variable given on a part of its Riemann surface in the form of a Puiseux series generated by the power function $z = w^{1/\rho}$, where $\rho > 1/2$ and $\rho \neq 1$. We present a many-sheeted variant of a theorem of G. Pólya describing the relation between the indicator and conjugate diagrams for entire functions of exponential type. The description is based on a construction of V. Bernstein for the many-sheeted indicator diagram of an entire function of order $\rho \neq 1$ and of normal type. The summation domain of a "proper" Puiseux series (a many-sheeted "Borel polygon") is found with the use of a generalization of the Borel method. This result seems to be new even in the case of power series. The theory applies to describe the domains of analytic continuation of Puiseux series representing the inverse functions for the rational ones. As but one consequence we elaborate a new approach to solution of algebraic equations.

Keywords: entire function, order, indicator, Puiseux series, multivalued function, many-sheeted diagram, concave diagram, indicator diagram, conjugate diagram, Riemann surface, analytic continuation, solution of algebraic equations.

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