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EXTREMAL SHIFT TO ACCOMPANYING POINTS IN A POSITIONAL DIFFERENTIAL GAME FOR A FRACTIONAL-ORDER SYSTEM

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A two-person zero-sum differential game is considered. The motion of the dynamical system is described by an ordinary differential equation with a Caputo fractional derivative of order $\alpha \in (0, 1)$. The performance index consists of two terms: the first depends on the motion of the system realized by the terminal time and the second includes an integral estimate of the realizations of the players' controls. The positional approach is applied to formalize the game in the "strategies — counter-strategies" and "counter-strategies — strategies" classes as well in the "strategies — strategies" class under the additional saddle point condition in the small game. In each case, the existence of the value and of the saddle point of the game is proved. The proofs are based on an appropriate modification of the method of extremal shift to accompanying points, which takes into account the specific properties of fractional-order systems.

Keywords: fractional-order differential equation, Caputo derivative, differential game, game value, positional strategy, counter-strategy, extremal shift.

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