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CONSTRUCTION OF A NONSMOOTH SOLUTION IN A TIME-OPTIMAL PROBLEM WITH A LOW ORDER OF SMOOTHNESS OF THE BOUNDARY OF THE TARGET SET

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Procedures for the construction of an optimal result function have been developed for a planar time-optimal control problem with a circular velocity vectorgram and nonconvex compact target set whose boundary has smoothness 1 or 2. Pseudovertrices, which are characteristic points of the boundary of the target set defining the character of the singularity of this function, are studied. Differential dependences for smooth segments of the singular set are revealed, which allows to consider and construct them as arcs of integral curves. The necessary conditions for the existence of pseudovertrices are found and formulas for the projections of points of the singular set in neighborhoods of pseudovertrices are obtained. The proposed procedures are implemented in the form of computational algorithms. Their efficiency is illustrated by examples of the numerical solution of optimal-time control problems with different orders of smoothness of the boundaries of the target sets. Visualization of the results is performed.

Keywords: time-optimal problem, singular set, dispersing curve, optimal result function, pseudo-vertex, symmetry set.

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