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EXAMPLE OF PARABOLIC SPLINE INTERPOLATION WITH BOUNDED LEBESGUE CONSTANT

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We consider an example of a sequence of geometric data grids for which the Lebesgue constant of interpolation by the classical parabolic splines (Subbotin's scheme) with periodic boundary conditions is unbounded; i.e., the interpolation process may diverge. We propose an alternative scheme for choosing the knots of a parabolic spline. In Subbotin's scheme, knots of a spline are chosen as the midpoints of intervals of the data grid, whereas the location of a knot in the alternative scheme is defined proportionally to the lengths of the adjacent intervals (we consider two variants). In the case of interpolation by the alternative scheme in the example under consideration, the process converges for any continuous function; i.e., the Lebesgue constant is bounded. The sequence of grids studied in the paper is the "worst" from the viewpoint of the convergence of the interpolation process in the classical case.

Keywords: parabolic splines, interpolation, convergence, Lebesgue constant.

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