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AN INEQUALITY OF DIFFERENT METRICS IN THE GENERALIZED LORENTZ SPACE

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The main goal of the paper is to prove the Jackson–Nikol'skii inequality for multiple trigonometric polynomials in the generalized Lorentz space $L_{\psi,\theta}(\mathbb{T}^m)$. In the first section we give definitions of a symmetric space of functions, a fundamental function, and the Boyd index of a space. In particular, we define the generalized Lorentz and Lorentz–Zygmund spaces. In addition, definitions of a weakly varying function and of the Lorentz–Karamata space are given. In the second section we prove an analog of the inequality of different metrics for multiple trigonometric polynomials in generalized Lorentz spaces $L_{\psi,\theta}(\mathbb{T}^m)$ with identical Boyd indices but different fundamental functions. In the Lorentz–Karamata space, the order-exact Jackson–Nikol'skii inequality for multiple trigonometric polynomials is obtained.

Keywords: Lorentz–Karamata space, Jackson–Nikol'skii inequality, trigonometric polynomial.

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