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BEST RESTRICTED APPROXIMATION OF SMOOTH FUNCTION CLASSES¹

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We first discuss the relative Kolmogorov n -widths of classes of smooth 2π -periodic functions for which the modulus of continuity of their r -th derivatives does not exceed a given modulus of continuity, and then discuss the best restricted approximation of classes of smooth bounded functions defined on the real axis \mathbb{R} such that the modulus of continuity of their r -th derivatives does not exceed a given modulus of continuity by taking the classes of the entire functions of exponential type as approximation tools. Asymptotic results are obtained for these two problems.

Keywords: modulus of continuity, best restricted approximation, average width.

REFERENCES

1. N. I. Achieser. *Theory of approximation*. N Y: Dover Publications, INC., 1992, 307 p. ISBN: 0486671291.
2. V. F. Babenko. Approximations in the mean with constraints on the derivatives of approximating functions. In : *Questions in Analysis and Approximations*. Kiev: Akad. Nauk Ukrain. SSR, Inst. Mat., 1989, pp. 9–18 (in Russian).
3. Dirong Chen. Average n -widths and optimal recovery of Sobolev classes in $L_p(\mathbb{R})$. *Chinese Ann. Math. Ser. B*, 1992, vol. 13, no. 4, pp. 396–405.
4. R. A. DeVore, G. G. Lorentz. *Constructive approximation*. Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 303, Berlin: Springer-Verlag, 1993, 449 p. ISBN: 3-540-50627-6.
5. R. S. Ismagilov On n -dimensional diameters of compacts in a Hilbert space. *Funct. Anal. Its. Appl.*, 1968, vol. 2, no. 2, pp. 125–132. doi: 10.1007/BF01075946.
6. Yanjie Jiang. *Widths and optimal recovery of smooth function classes*. PhD thesis, Beijing Normal University, 1998.
7. V. N. Konovalov. Estimates of Kolmogorov-type widths for classes of differentiable periodic functions. *Math. Notes*, 1984, vol. 35, no. 3, pp. 193–199. doi: 10.1007/BF01139916.
8. V. N. Konovalov. Approximation of Sobolev classes by their finite-dimensional sections. *Math. Notes*, 2002, vol. 72, no. 3, pp. 337–349. doi: 10.1023/A:1020547320561.
9. V. N. Konovalov. Approximation of Sobolev classes by their sections of finite dimension. *Ukraine Math. J.*, 2002, vol. 54, no. 5, pp. 795–805. doi: 10.1023/A:1021635530578.
10. N. P. Korneichuk. *Ekstremal'nye zadachi teorii priblizheniya* [Extremal problems of approximation theory]. Moscow: Nauka Publ., 1976, 320 p.
11. N. P. Korneichuk. *Exact constants in approximation theory*. Cambridge: Cambridge University Press, 1991, Encyclopedia Math. Appl., vol. 38, 466 p. ISBN: 9781107094277.
12. Bo Ling, Yongping Liu. Best restriction approximation of Sobolev classes by entire functions of exponential type. *Acta Mathematica Sinica, Chinese Series*, 2017, vol. 60, no. 3, pp. 389–400.
13. Yongping Liu. *Infinite dimensional widths and optimal recovery on the $S - W$ spaces*. PhD thesis, Beijing Normal University, 1993.

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14. Yongping Liu, Weiwei Xiao. Relative average widths of Sobolev spaces in $L_2(\mathbb{R}^d)$. *Anal. Math.*, 2008, vol. 34, no. 1, pp. 71–82. doi: 10.1007/s10476-008-0107-8.
15. G. G. Lorentz, M. V. Golitschek, Y. Makovoz *Constructive approximation. Advanced problems.* Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences], vol. 304. Berlin: Springer-Verlag, 1996, 649 p. ISBN: 3-540-57028-4.
16. G. G. Magaril-Il'yaev. φ -mean diameters of classes of functions on the line. *Russian Math. Surv.*, 1990, vol. 45, no. 2, pp. 218–219. doi: 10.1070/RM1990v045n02ABEH002340.
17. G. G. Magaril-Il'yaev. Mean dimension, widths, and optimal recovery of Sobolev classes of functions on the line. *Math. USSR-Sb.*, 1993, vol. 74, no. 2, pp. 381–403. doi: 10.1070/SM1993v074n02ABEH003352.
18. G. G. Magaril-Il'yaev, V. M. Tikhomirov. Average dimension and ν -widths of classes of functions on the whole line. *J. Complexity*, 1992, vol. 8, no. 1, pp. 64–71. doi: 10.1016/0885-064X(92)90034-9.
19. S. M. Nikol'skii. *Approximation of functions of several variables and embedding theorems.* Berlin; N Y: Springer-Verlag, 1975, 420 p. ISBN: 0387064427.
20. Heping Wang. *Approximation and quadrature formula on function classes with mixed smoothness.* PhD thesis, Beijing Normal University, 1996.
21. Yongsheng Sun, Yongping Liu, Dirong Chen. Extremal problems in approximation theory for some classes of smooth functions defined on \mathbb{R}^d . *J. Beijing Normal University (Natural Sciences)*, 1999, vol. 35(supp.), pp. 79–144.
22. V. M. Tikhomirov. Some remarks on relative diameters. In: *Approximation and function spaces, Proc. 27th Semest., Warsaw/Pol. 1986, Banach Cent. Publ.*, 1989, vol. 22, pp. 471–474.
23. V. M. Tikhomirov. On approximation properties of smooth functions. *Proc. Conf. on Differential Equations and Numerical Mathematics, (Novosibirsk: Nauka Publ., 1980)*, pp. 183–188 (in Russian).
24. Guiqiao Xu. *Widths and optimal recovery of multivariate smooth functions.* PhD thesis, Beijing Normal University, 2001.
25. Wei Yang. *Relative widths of differentiable function classes and convolution classes with 2π periodic in one variable case and hexagonal periodic in 2-dimensional case.* PhD thesis, Beijing Normal University, 2009.

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