RELAXATION OF THE PURSUIT–EVASION DIFFERENTIAL GAME AND ITERATIVE METHODS

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A variant of the program iteration method called stability iterations is used for a differential game of pursuit–evasion. The successful solvability set of one of the problems generating the game is found as a limit of the iterative procedure in the space of sets whose elements are positions of the game. The game is defined by a pair of closed sets, one of the which is the objective set in the pursuit problem (the first player's problem) and the other specifies the state constraints in this problem. For the positions not belonging to the solvability set of the pursuit problem, it is interesting to determine the smallest "size" of a neighborhood of the two mentioned sets for which the first player can implement the guidance to the neighborhood of the objective set corresponding to this "size" within the similar neighborhood of the second set, i.e., the set specifying the state constraints. Similar constructions are considered for the sets realized at each stage of the iterative procedure. We use the connection of these constructions with the mentioned smallest "size" of neighborhoods of the sets that are parameters of the differential game in the sense of guaranteed realizability of guidance under the replacement of the original sets by these neighborhoods.

Keywords: differential game of pursuit–evasion, program iteration method, guaranteed guidance.

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