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## POINTWISE TURÁN PROBLEM FOR PERIODIC POSITIVE DEFINITE FUNCTIONS

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We study the pointwise Turán problem on the largest value at an arbitrary point  $x$  of a 1-periodic positive definite function supported on the interval  $[-h, h]$  and equal to 1 at zero. For rational values of  $x$  and  $h$ , the problem reduces to a discrete version of the Fejér problem on the largest value of the  $\nu$ th coefficient of an even trigonometric polynomial of order  $p - 1$  that has zero coefficient 1 and is nonnegative on a uniform grid  $k/q$ ,  $k = 0, \dots, q - 1$ . The discrete Fejér problem is solved for a number of values of the parameters  $\nu$ ,  $p$ , and  $q$ . In all the cases, we construct extremal polynomials and quadrature formulas, which yield an estimate for the largest coefficient.

Keywords: Fourier transform and series, periodic positive definite function, pointwise Turán problem, quadrature formula, extremal polynomial.

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