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## The MAL'TSEV CORRESPONDENCE AND ISOMORPHISMS OF NILTRIANGULAR SUBRINGS OF CHEVALLEY ALGEBRAS

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Models of algebraic systems of a first-order language are called elementarily equivalent (we write  $\equiv$ ) if every sentence that is true in one of the models is also true in the other model. The model-theoretic study of linear groups and rings initiated by A.I. Mal'tsev (1960, 1961) is closely related to isomorphism theory; as a rule, the relation  $\equiv$  of systems was transferred to fields (or rings encountered) of the coefficients. The Mal'tsev correspondence was analyzed for rings of niltriangular matrices and unitriangular groups (B. Rose, 1978; V. Weiler, 1980; K. Videla, 1988; O. V. Belegradek, 1999; V. M. Levchuk, E. V. Minakova, 2009). For unipotent subgroups of Chevalley groups over a field  $K$ , the correspondence was studied in 1990 by Videla for  $\text{char } K \neq 2, 3$ . Earlier the authors announced a weakening of the constraint on the field  $K$  in the Videla theorem. In the Chevalley algebra associated with a root system  $\Phi$  and a ring  $K$ , the niltriangular subalgebra  $N\Phi(K)$  is naturally distinguished. The main results of this paper establish the Mal'tsev correspondence (related with the description of isomorphisms) for the Lie rings  $N\Phi(K)$  of classical types over arbitrary associative commutative rings with unity. A corollary is noted for (nonassociative) enveloping algebras to  $N\Phi(K)$ .

Keywords: Chevalley algebra, niltriangular subalgebra, isomorphism, model-theoretic Mal'tsev correspondence.

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