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ON INTERSECTIONS OF NILPOTENT SUBGROUPS IN FINITE GROUPS WITH SOCLE $L_2(2^m) \times L_2(2^n)$

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In Theorem 1, it is proved for a finite group G with socle $L_2(2^m) \times L_2(2^n)$ and nilpotent subgroups A and B that the condition $\min_G(A, B) \neq 1$ implies that $n = m = 2$ and the subgroups A and B are 2-groups. Here the subgroup $\min_G(A, B)$ is generated by smallest-order intersections of the form $A \cap B^g$, $g \in G$, and the subgroup $\mathrm{Min}_G(A, B)$ is generated by all intersections of the form $A \cap B^g$, $g \in G$, that are minimal with respect to inclusion. In Theorem 2, for a finite group G with socle $A_5 \times A_5$ and a Sylow 2-subgroup S, we give a description of the subgroups $\min_G(S, S)$ and $\mathrm{Min}_G(S, S)$. On the basis of Theorem 2, in Theorem 3 for a finite group G with socle $A_5 \times A_5$ we describe up to conjugation all pairs of nilpotent subgroups (A, B) of G for which $\min_G(A, B) \neq 1$.

Keywords: finite groups, nilpotent subgroup, intersection of subgroups.

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